Notes on scheduling radio-astronomical observations

Maher Mallem[1], Frédéric Vivien[1]

[1]Inria, CNRS, ENS de Lyon, Université Claude Bernard Lyon 1, LIP, UMR 5668, 69342, Lyon cedex 07, France

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Outline

Outline

Observing from the Earth

Figure: Time lapse photography (Animalia Life)

Altitude over time

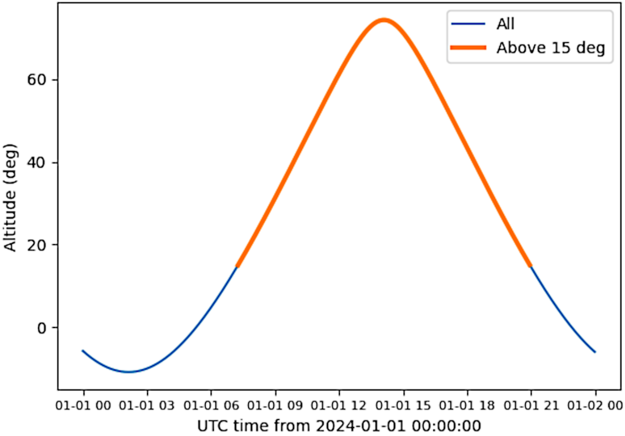

Figure: Altitude of star KELT 16 from the NenuFAR radio-telescope on 1 January 2024.

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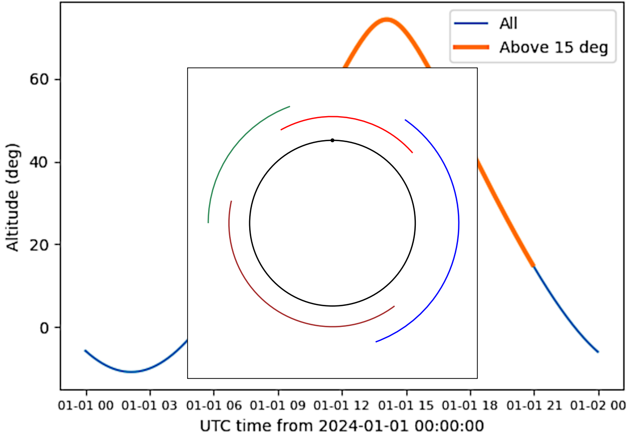

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Postdoc: "Observation Scheduling"

ROMA team, LIP, ENS de Lyon Nov. 2024 - Now

- *Topic:* Scheduling radio-astronomical observations.
  - Joint work with Frédéric Vivien and Anne Benoit.
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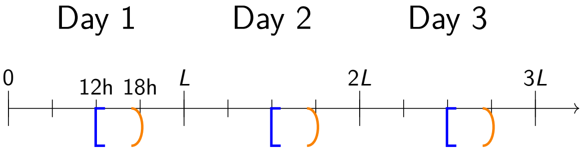
  - Processing time $p_j \in \mathbb{R}_{>0}$, daily release time $r_j \in [0, L)$ and daily deadline $d_j \in [0, L)$.



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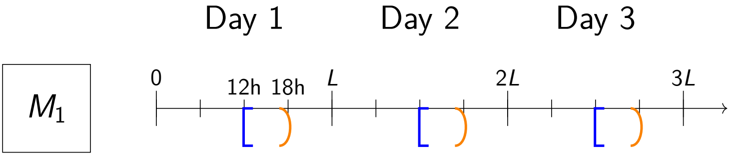
  - Single machine, non-preemptive, minimizing idle time (i.e., makespan).



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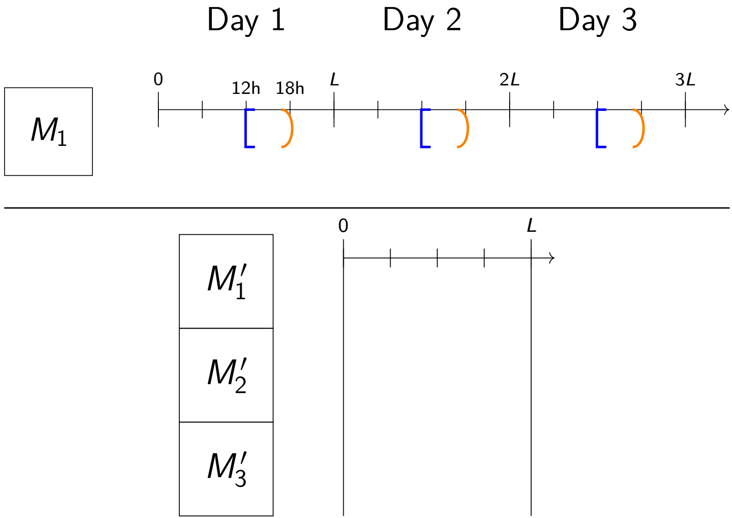

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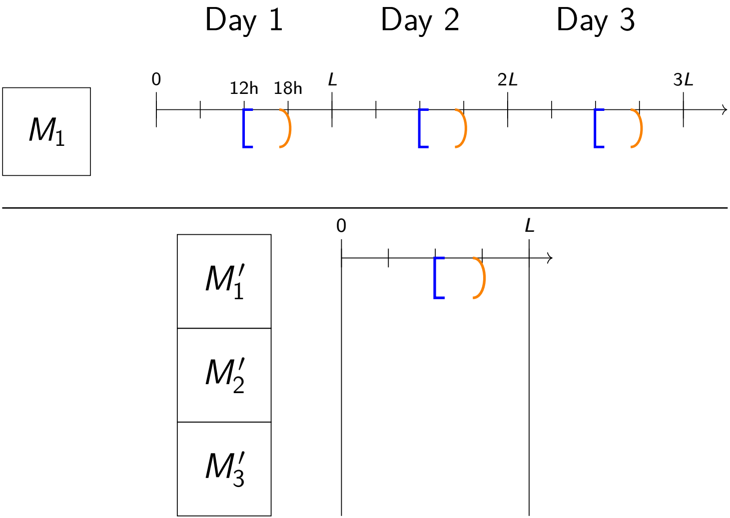

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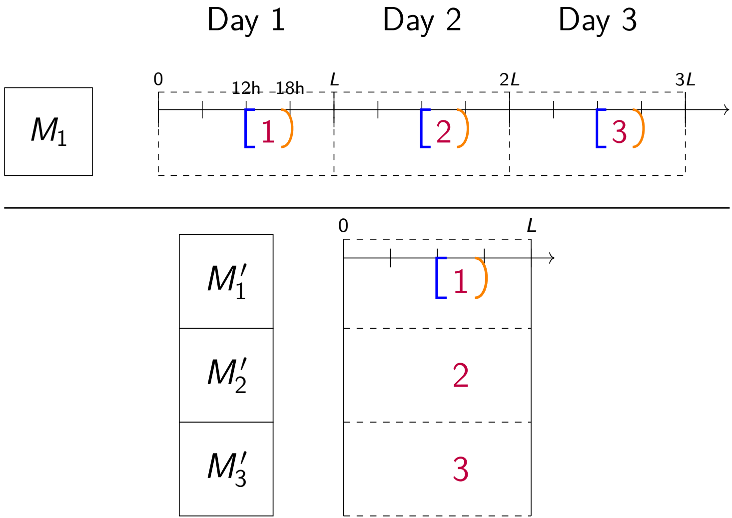

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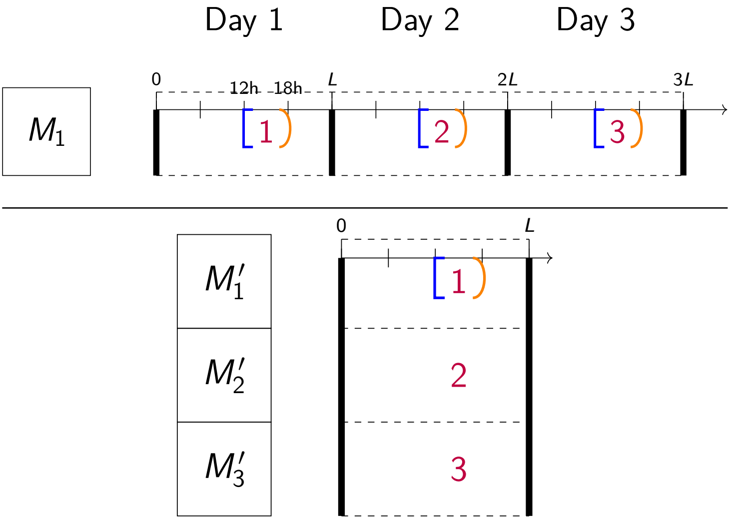

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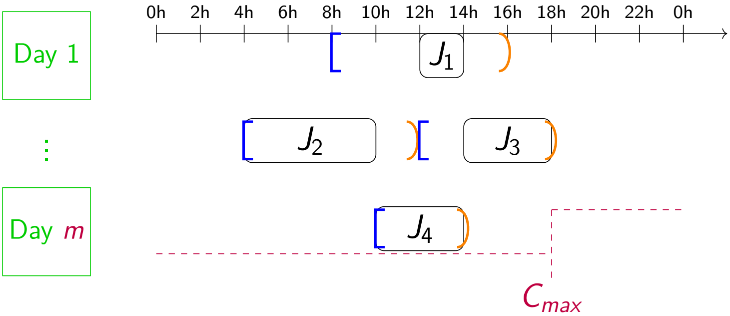

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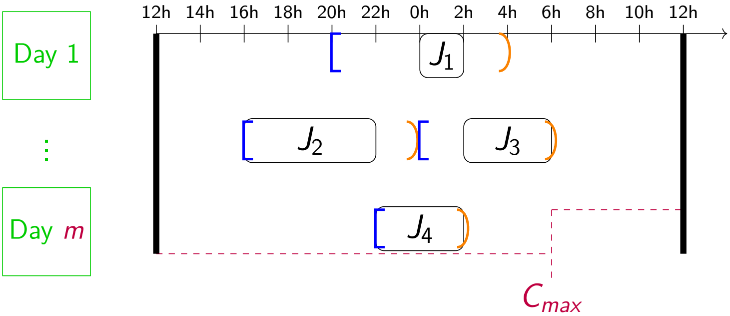

With isolated days: $P|r_j, d_j|\min(m)$.

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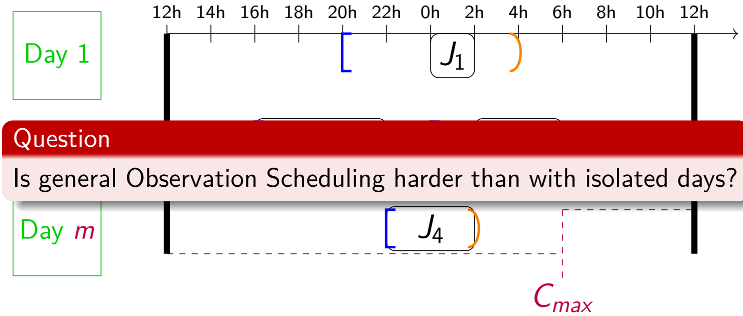
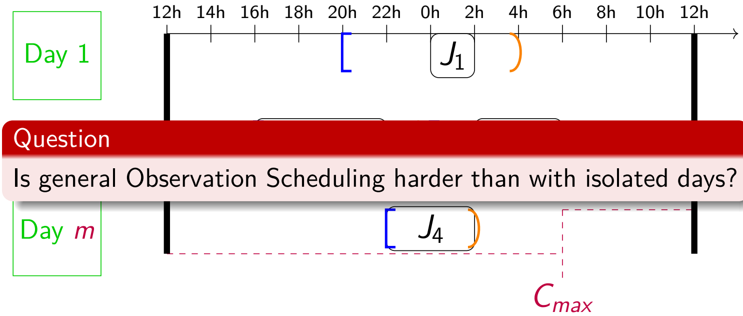


Question

Is general Observation Scheduling harder than with isolated days?

With isolated days: $P|r_j, d_j|\min(m)$.



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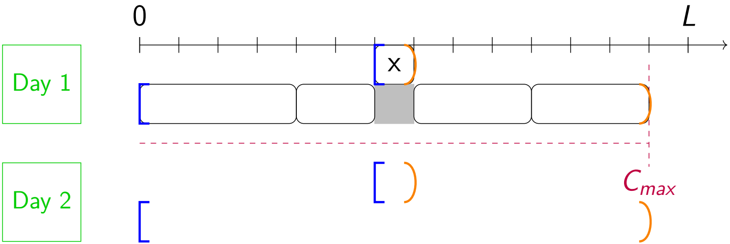
One day: strongly NP-complete [Lenstra et al., 1977].

Outline

$(2-\varepsilon)$-inapproximability



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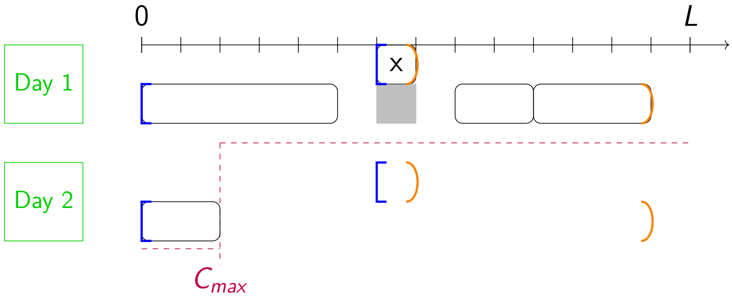

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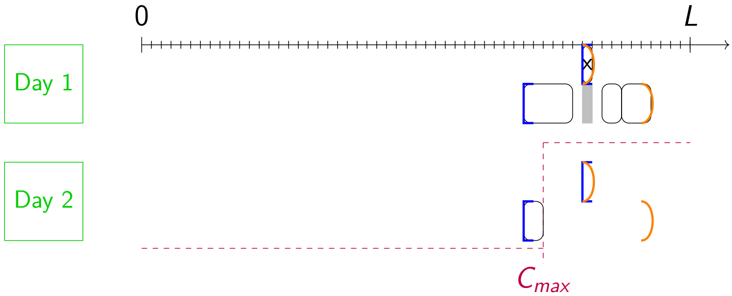

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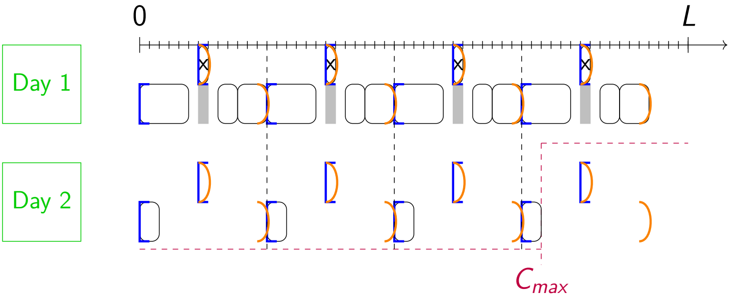

Observation Scheduling with isolated days.

- Goal: minimize the number of days.
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Job Interval Selection problem.

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- 1/2-approximation: "Earliest Completion Time". [Spieksma, 1999]

$\mathcal{O}(\log(n))$-approximation (cont.)

Remark

W.l.o.g: $\forall j, p_j \leq L$.

Proposition

If $L \leq OPT$, then any f-approximation algorithm with isolated days is a $[(2 + L/OPT) \cdot f]$-approximation algorithm for Observation Scheduling.

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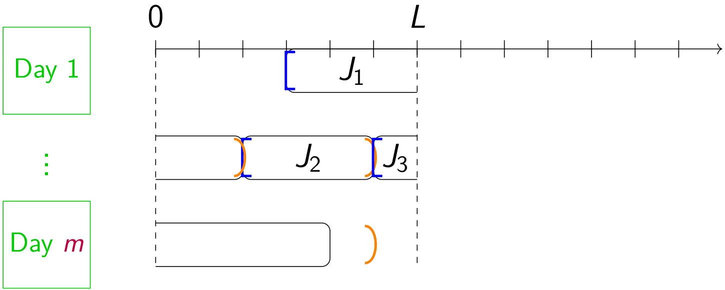

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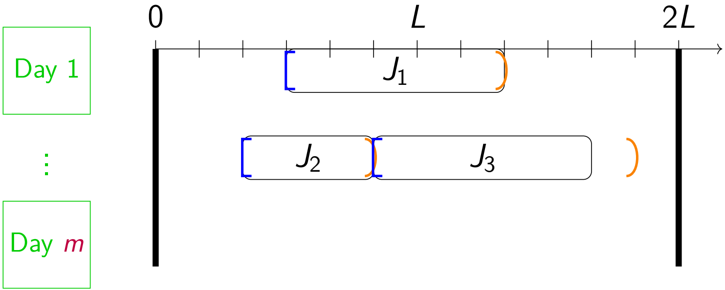

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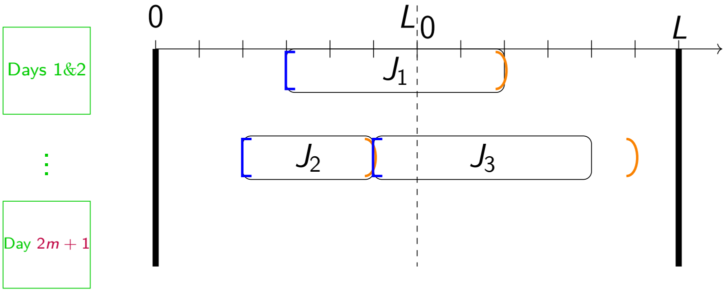

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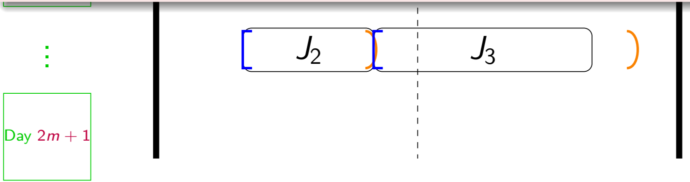

Outline

Going beyond NP-hardness

- Problem $\mathcal{P}$, instance $\mathcal{I}$.
- P: deterministic time $\textit{poly}(|\mathcal{I}|)$.
- NP: nondeterministic time $\textit{poly}(|\mathcal{I}|)$.

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- Problem $\mathcal{P}$, instance $\mathcal{I}$.
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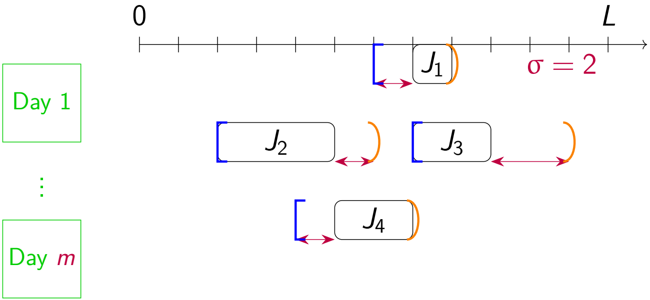
  - para-NP-complete: NP-complete for some fixed k.

Scheduling with tight windows



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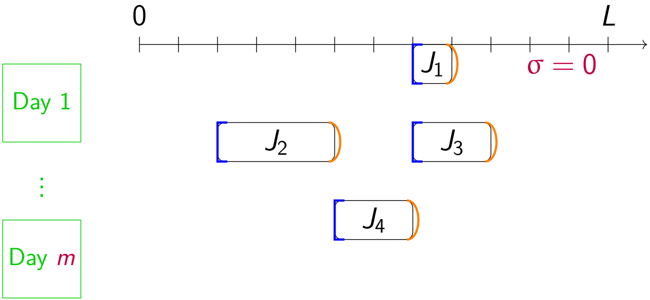

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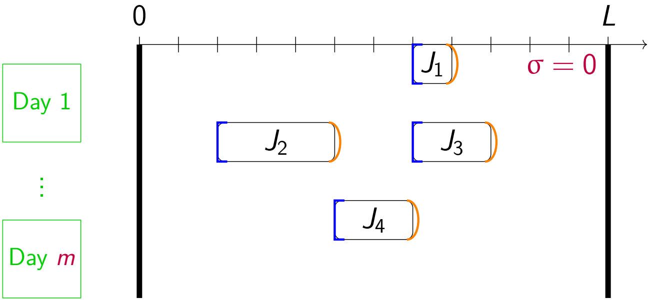

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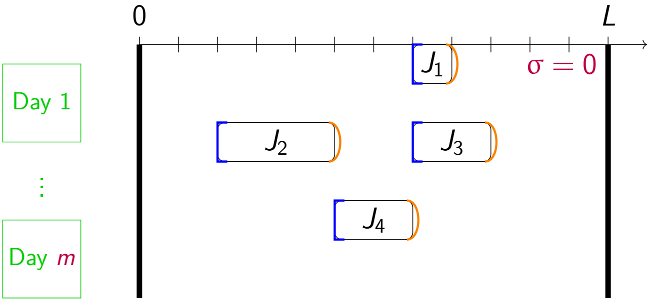

Interval graph coloring: $\mathcal{O}(n \cdot \log(n))$ time.

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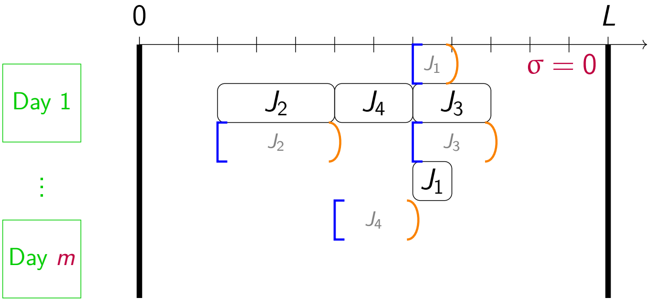

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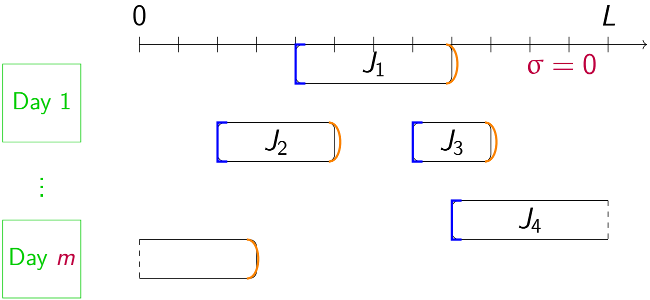

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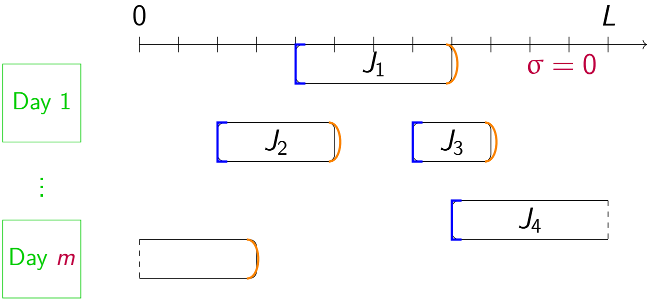
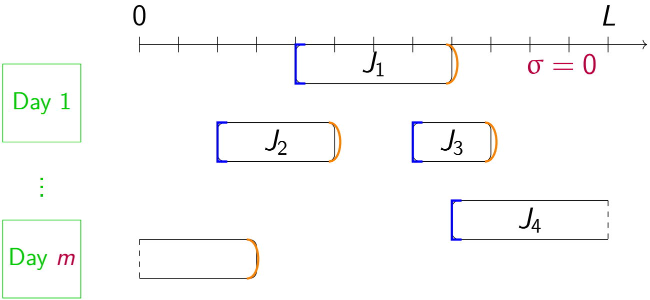


Circular-arc graph coloring: NP-complete [Garey et al., 1980].



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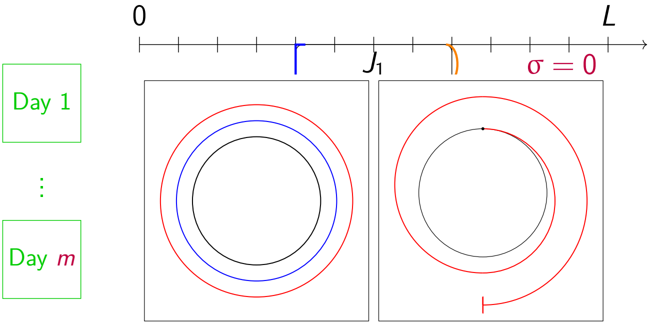

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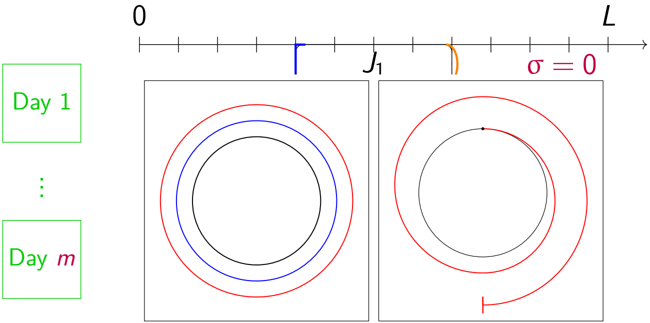

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Multi-slot Just-in-time: $\mathcal{O}(n \log(n)^2)$ time [Dereniowski and Kubiak, 2010].

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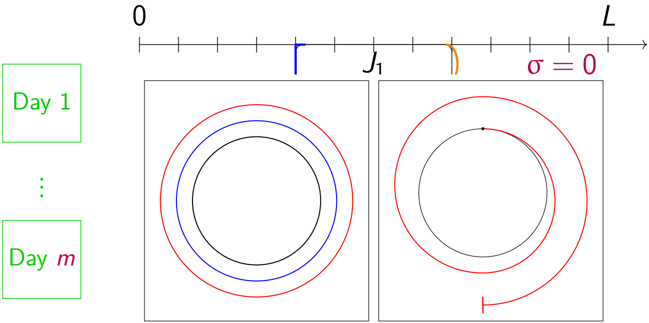

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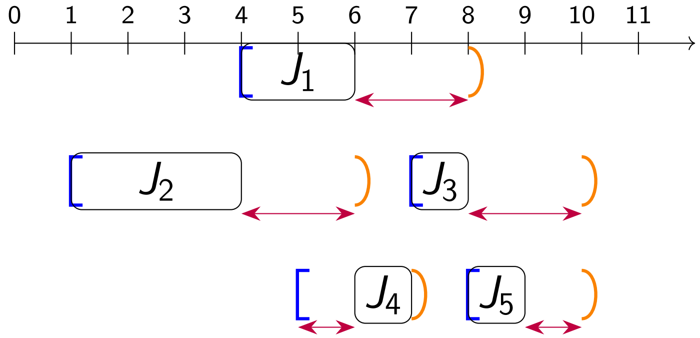
$\rightarrow$ TSP with a Gilmore-Gomory distance matrix.

[Gilmore and Gomory, 1964, Vairaktarakis, 2003]

Width μ



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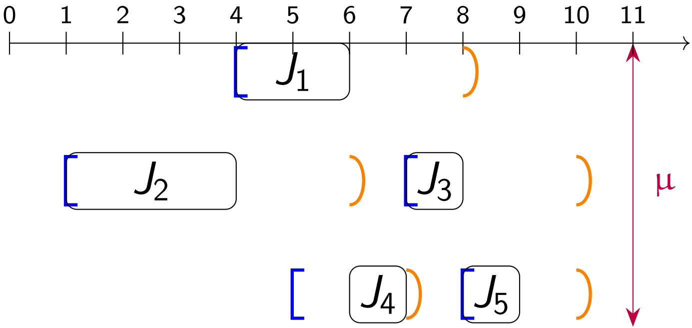

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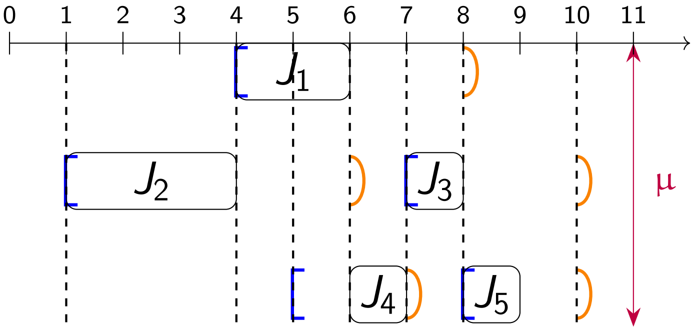

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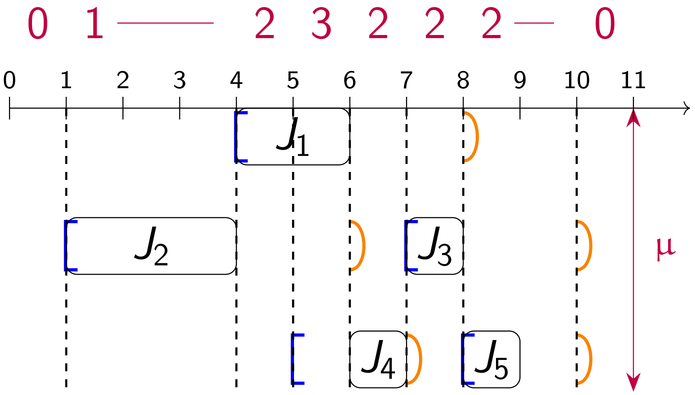

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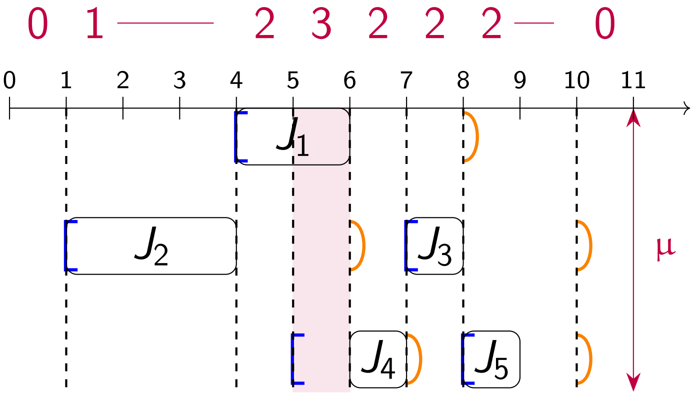

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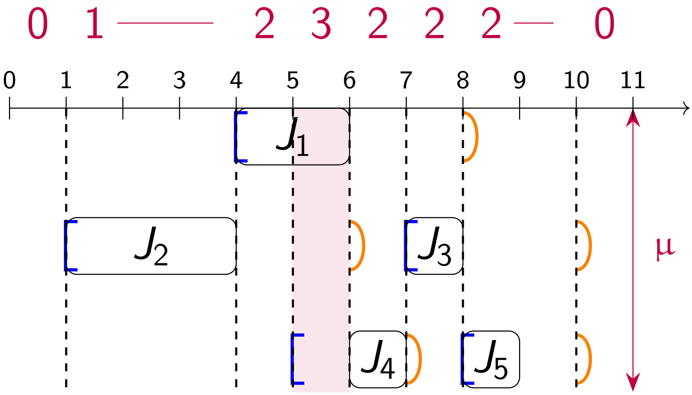

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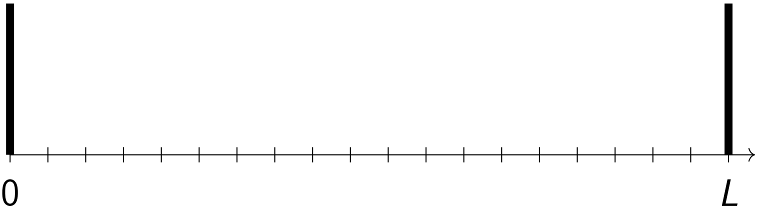
FPT membership with $(\mu + \sigma)$: border schedules



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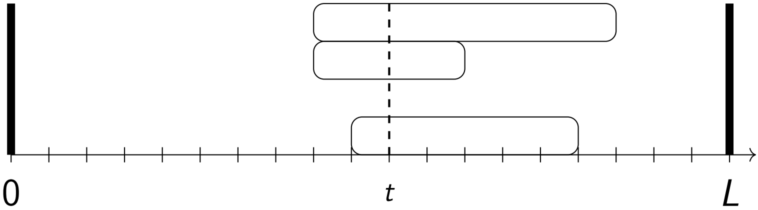

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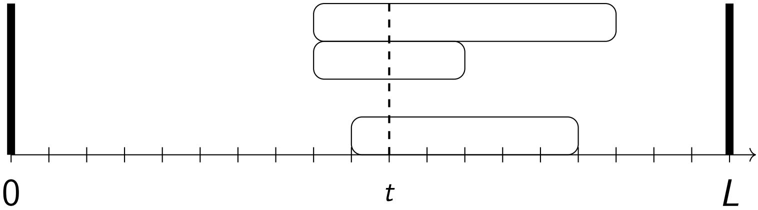

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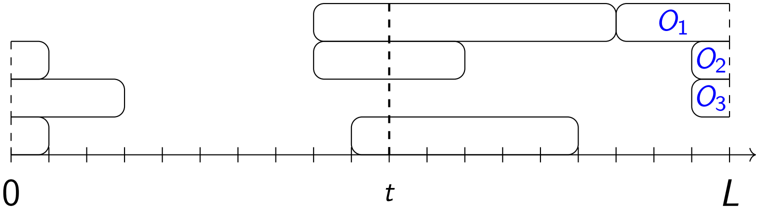

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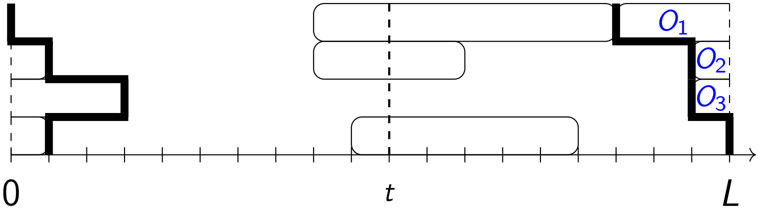

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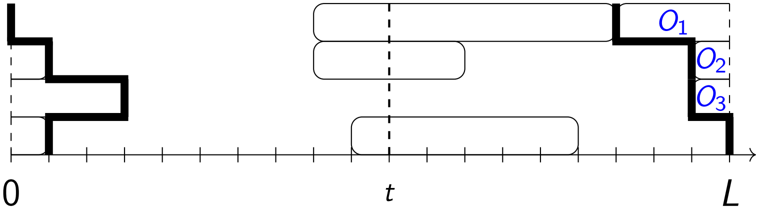

- Isolated days: in $\mathrm{FPT}(\mu + \sigma)$.
  [Cieliebak et al., 2004, Hanen and Munier Kordon, 2023]
- $\mathcal{O}([\mu(\sigma+1)+1]^m)$ border schedules associated to t mod L.

Idea

For each of the $\mathcal{O}([\mu(\sigma+1)+1]^{m-1})$ border schedules associated to 0 mod L, solve an instance of "$P|r_j, d_j|\min(m)$ with machine availabilities".

Outline

Conclusion

Results: same as with isolated days

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- No $(2-\varepsilon)$-approximation algorithm (unless P = NP).
- Fixed-parameter tractability: σ, μ and $(m+\sigma)$.

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Future work

- Constant approximation algorithm?
- Improve the fixed-parameter algorithms.
  - $\mathcal{O}([\mu(\sigma+1)+1]^{3m-1} \cdot 2^{3\mu} \cdot m^{\mu} \cdot \mu \cdot n + n \cdot \log(n))$ time.
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- **Other applications?**

References I

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