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Computational complexity of radio-astronomical observation scheduling

Maher Mallem[1], Frédéric Vivien[1]

[1]Inria, CNRS, ENS de Lyon, Université Claude Bernard Lyon 1, LIP, UMR 5668, 69342, Lyon cedex 07, France

25 February 2026

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Observing from the Earth

Figure: Time lapse photography (Animalia Life)

Altitude over time

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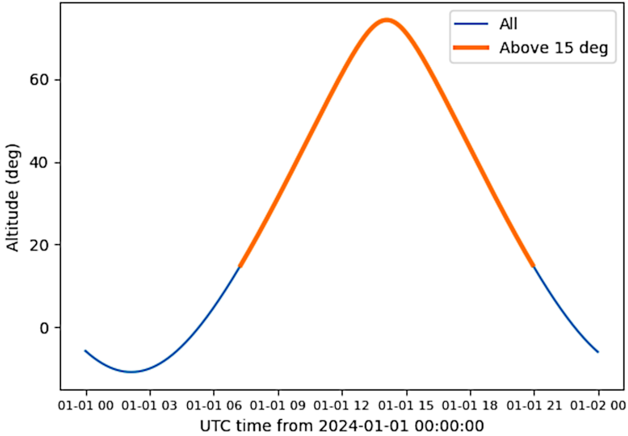

Figure: Altitude of star KELT 16 from the NenuFAR radio-telescope on 1 January 2024.

Altitude over time

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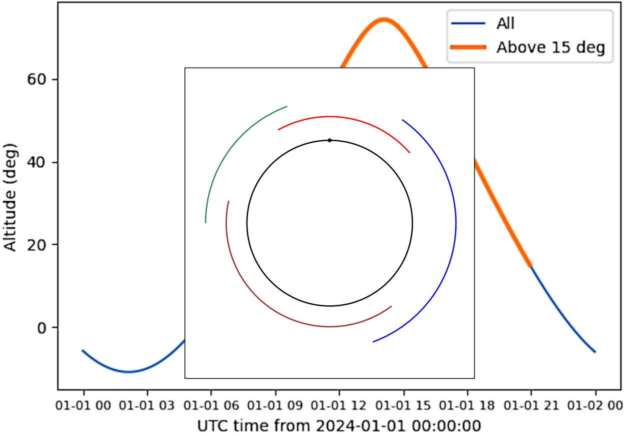

Figure: Altitude of star KELT 16 from the NenuFAR radio-telescope on 1 January 2024.

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Postdoc: "Observation Scheduling"

ROMA team, LIP, ENS de Lyon Nov. 2024 - April 2026

- *Topic:* Scheduling radio-astronomical observations.
  - Joint work with Frédéric Vivien.
  - Partnership with the ECLAT laboratory (HPC & AI for astronomy & astrophysics).
  - Access to NenuFAR radio-telescope usage data.

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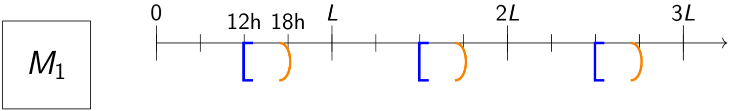
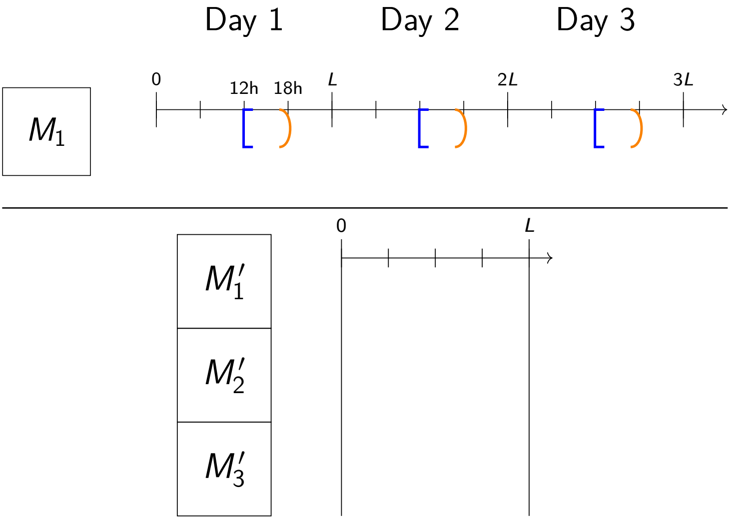
  - Joint work with Frédéric Vivien.
  - Partnership with the ECLAT laboratory (HPC & AI for astronomy & astrophysics).
  - Access to NenuFAR radio-telescope usage data.
- *Model:* **"Observation Scheduling"**.
  - Day length $L \in \mathbb{R}_{>0}$.
  - Processing time $p_j \in \mathbb{R}_{>0}$, daily release date $r_j \in [0, L)$ and daily deadline $d_j \in [0, L)$.
  - Single machine, non-preemptive.

Day 1 Day 2 Day 3



Postdoc: "Observation Scheduling" (cont.)



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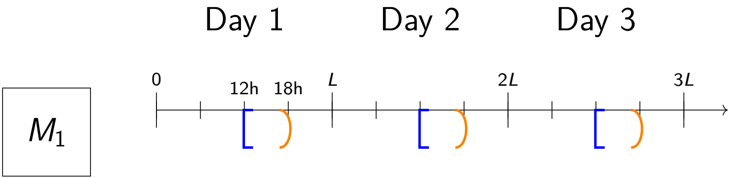

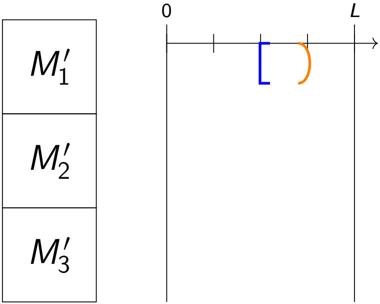

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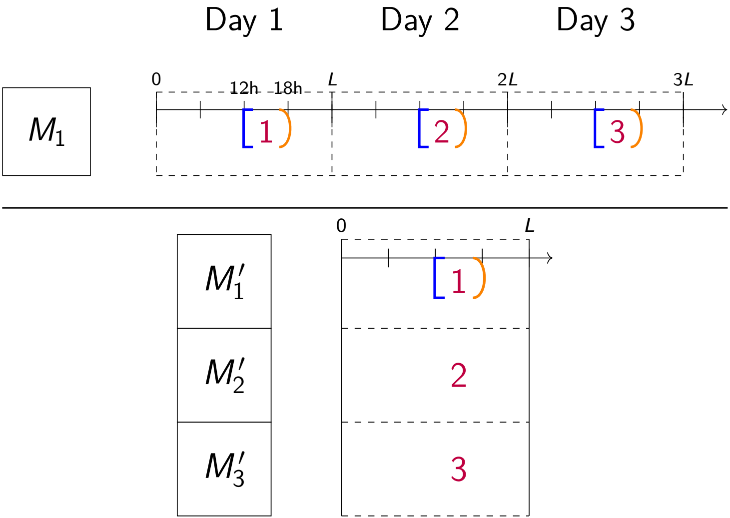

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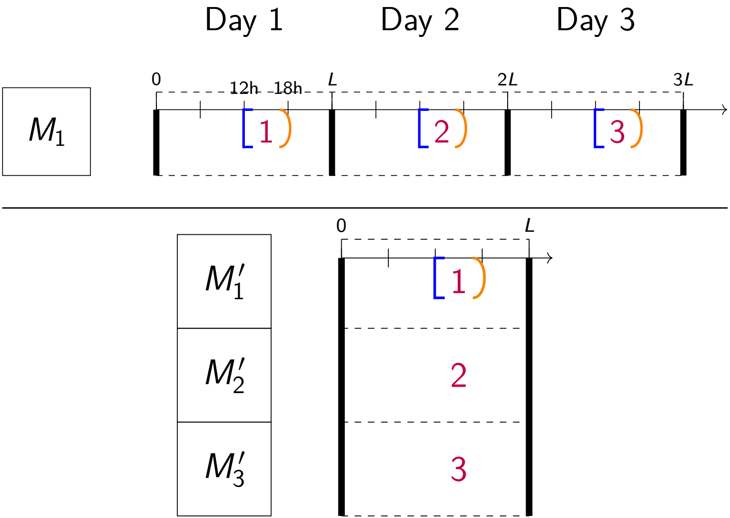

Scheduling observations

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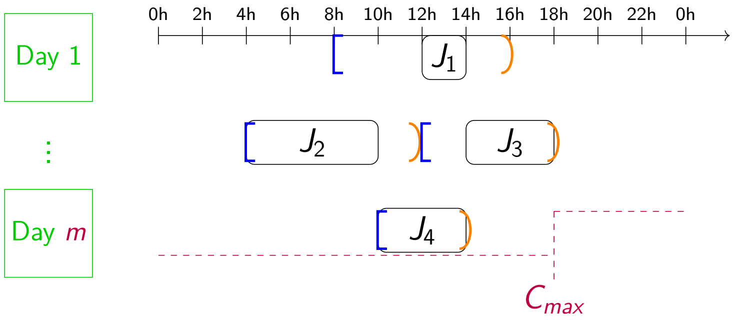

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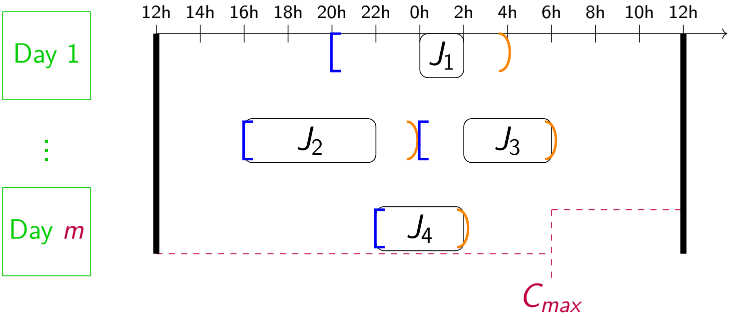

With isolated days: $P|r_j, d_j|\min(m)$.

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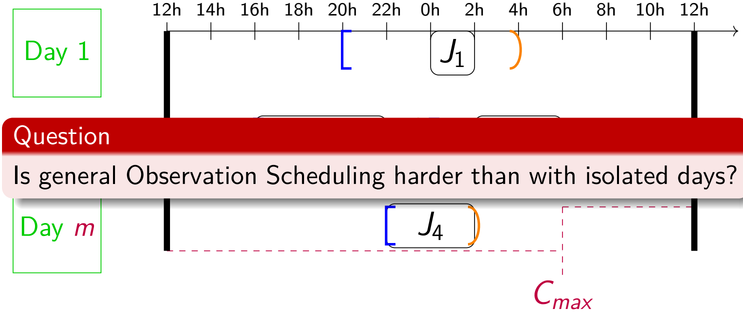
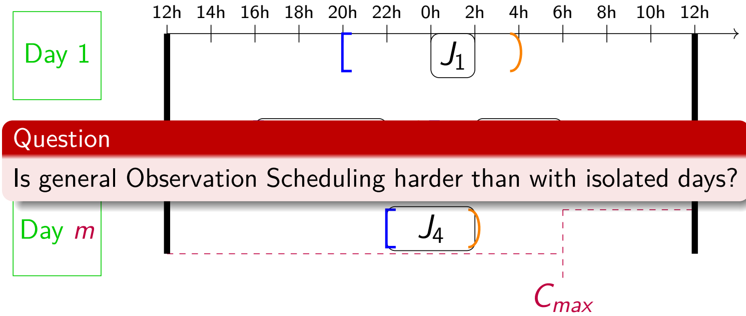


Question

Is general Observation Scheduling harder than with isolated days?

With isolated days: $P|r_j, d_j|\min(m)$.

Scheduling observations



Question

Is general Observation Scheduling harder than with isolated days?

With isolated days: $P|r_j, d_j|\min(m)$.

One day: strongly NP-complete [Lenstra et al., 1977].

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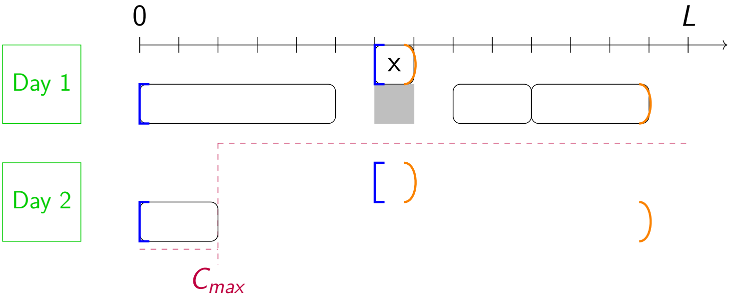

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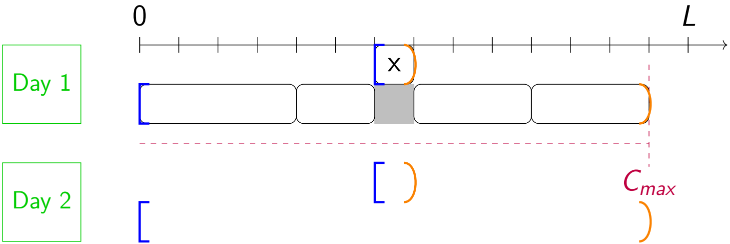

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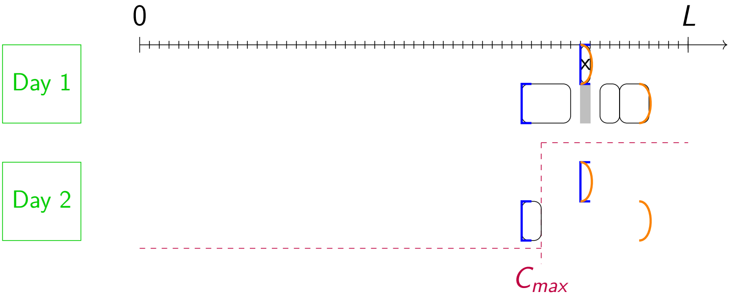

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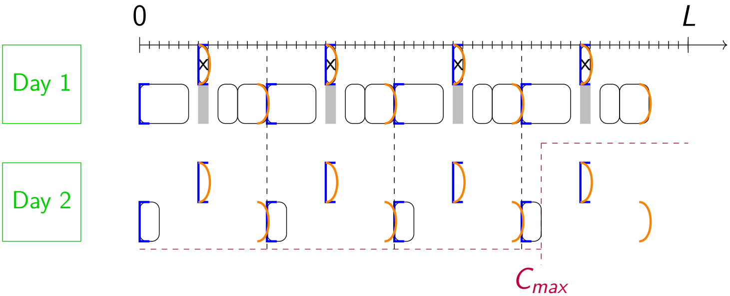

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Observation Scheduling with isolated days.

- Goal: minimize the number of days.
  - $P|r_j, d_j|\min(m)$.

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Job Interval Selection problem.

- Goal: maximize the number of scheduled jobs.

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Job Interval Selection problem.

- Goal: maximize the number of scheduled jobs.
- 1/2-approximation: "Earliest Completion Time". [Spieksma, 1999]

$\mathcal{O}(\log(n))$-approximation (cont.)

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Remark

W.l.o.g: $\forall j, p_j \leq L$.

Proposition

If $L \leq OPT$, then any f-approximation algorithm with isolated days is a $[(2 + L/OPT) \cdot f]$-approximation algorithm for Observation Scheduling.

$\mathcal{O}(\log(n))$-approximation (cont.)

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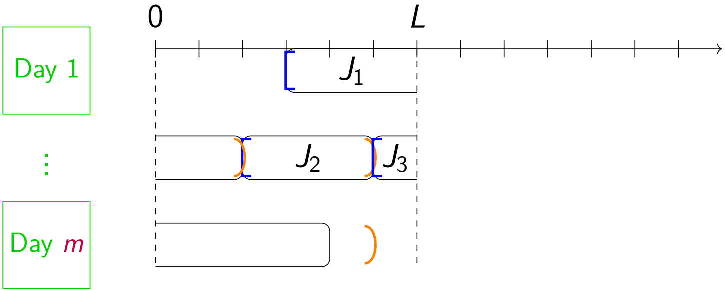

$\mathcal{O}(\log(n))$-approximation (cont.)

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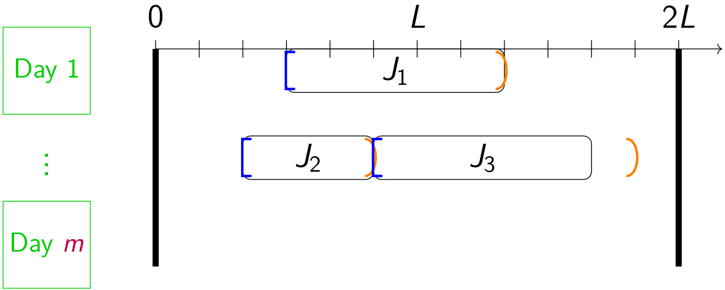

$\mathcal{O}(\log(n))$-approximation (cont.)

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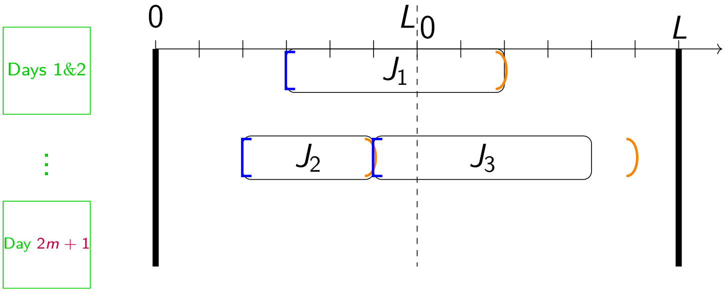

$\mathcal{O}(\log(n))$-approximation (cont.)

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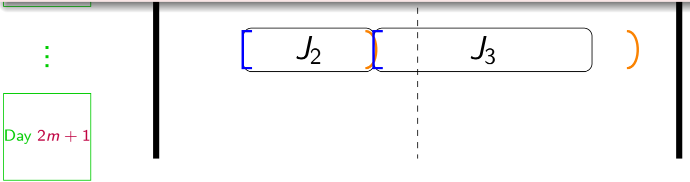
Question

Constant approximation algorithm?



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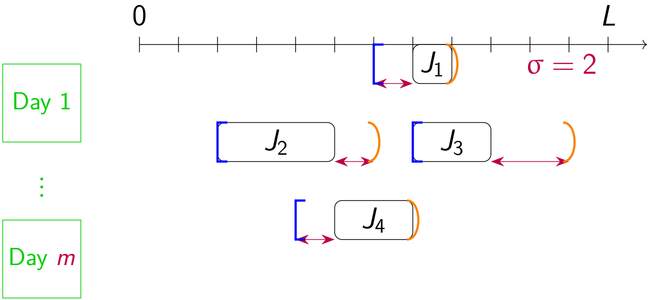

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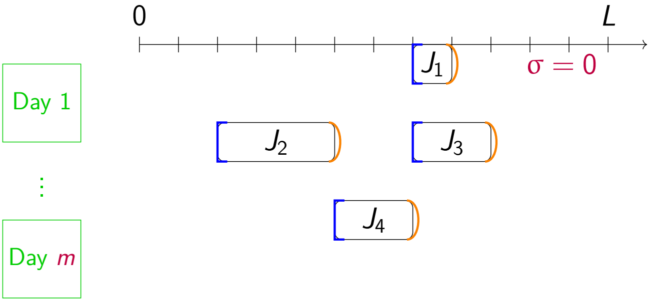

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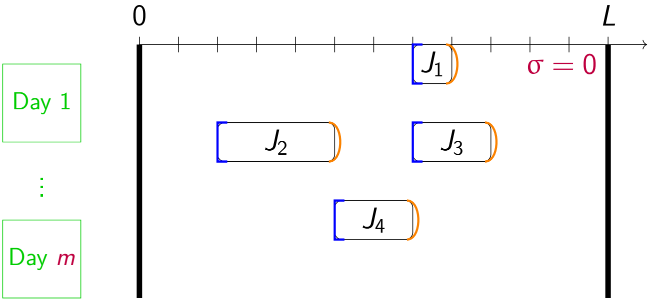

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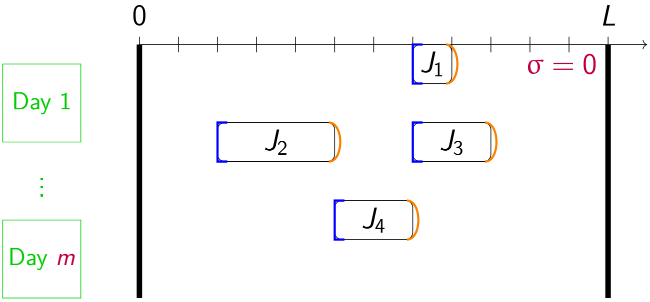

Interval graph coloring: $\mathcal{O}(n \cdot \log(n))$ time.

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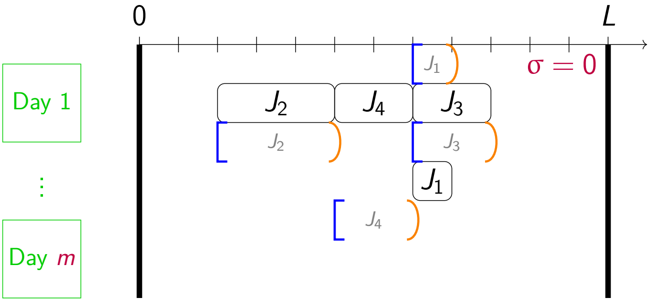

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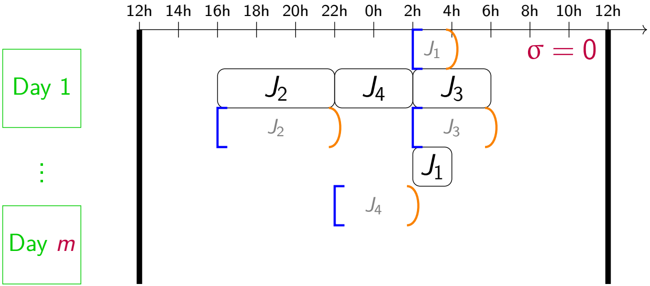

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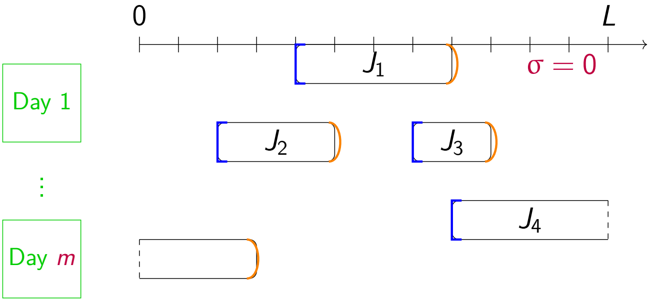

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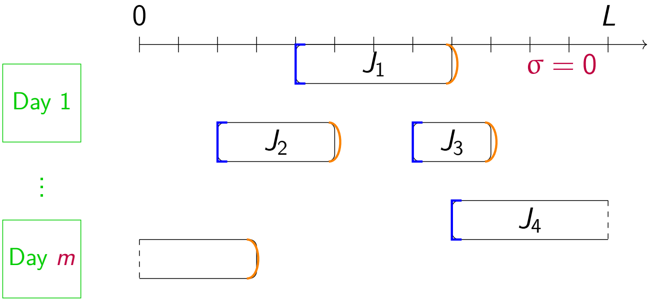

Circular-arc graph coloring: NP-complete [Garey et al., 1980].

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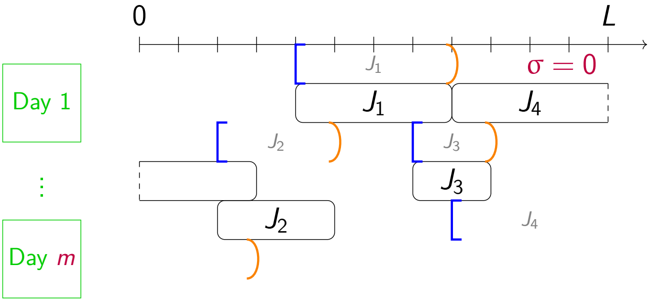

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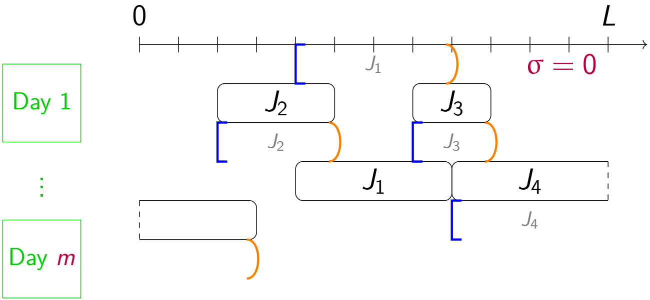

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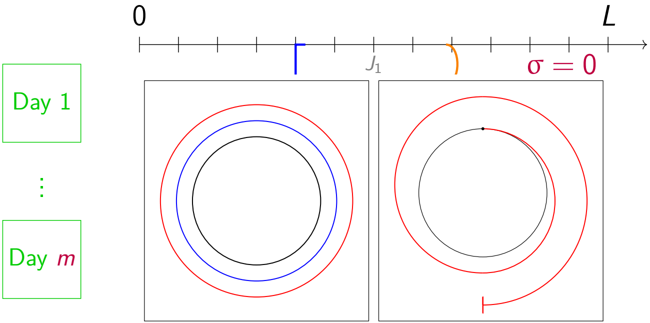

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Multi-slot Just-in-time: $\mathcal{O}(n \log(n)^2)$ time [Dereniowski and Kubiak, 2010].

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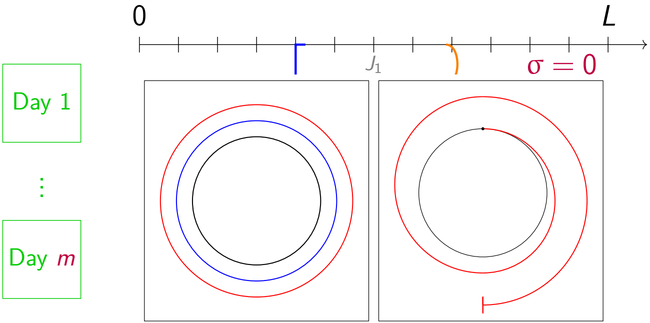

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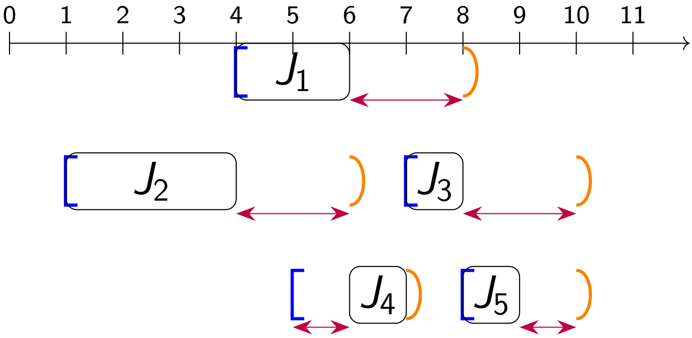
Multi-slot Just-in-time: $\mathcal{O}(n \log(n)^2)$ time [Dereniowski and Kubiak, 2010].

$\rightarrow$ TSP with a Gilmore-Gomory distance matrix.

[Gilmore and Gomory, 1964, Vairaktarakis, 2003]

Width μ



$\sigma = 2$: NP-complete [Cieliebak et al., 2004].

Width μ

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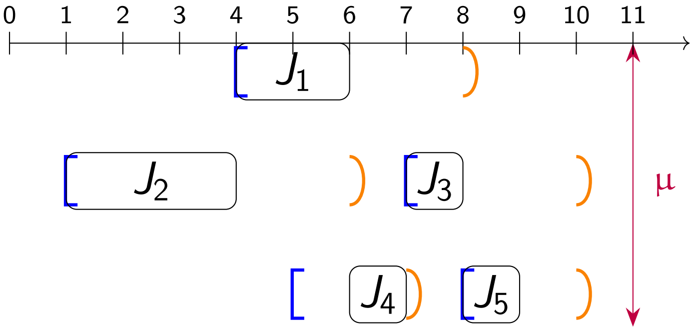

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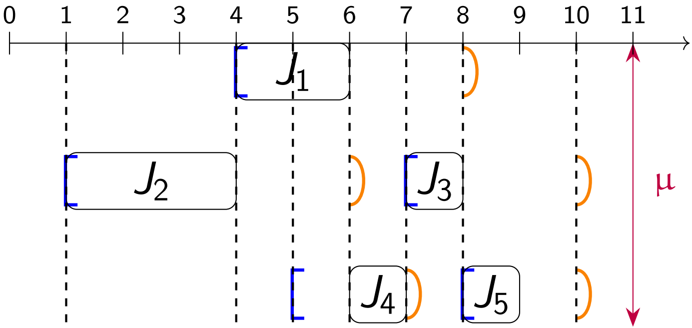

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Width μ

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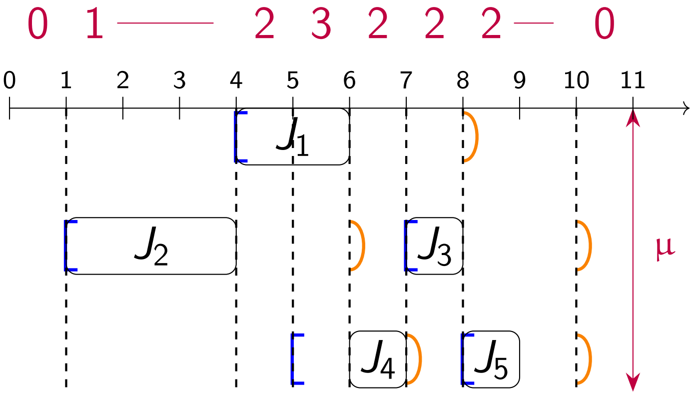

$\sigma = 2$: NP-complete [Cieliebak et al., 2004].

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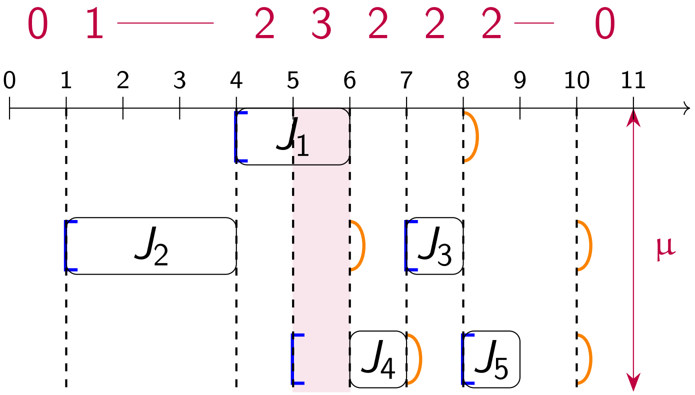

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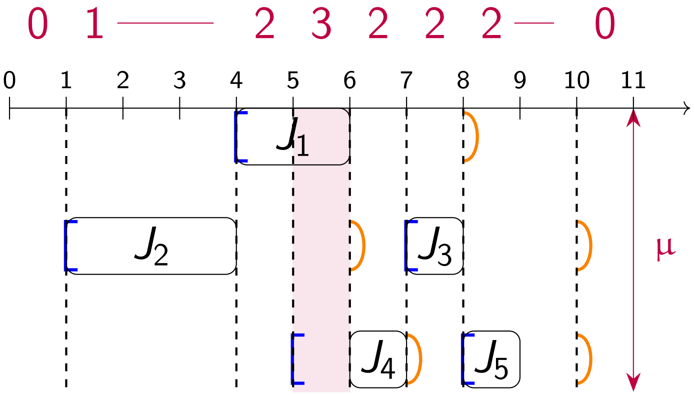

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$\mu = 4$: NP-complete [Hanen and Munier Kordon, 2023].

Border schedules

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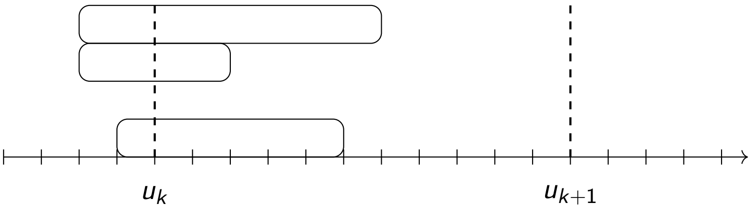

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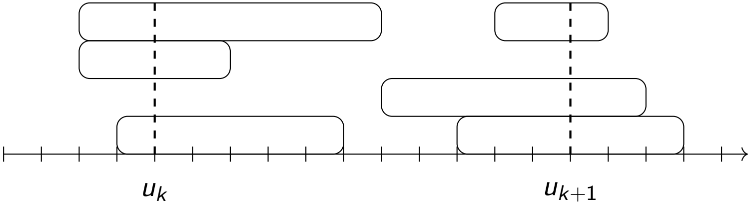

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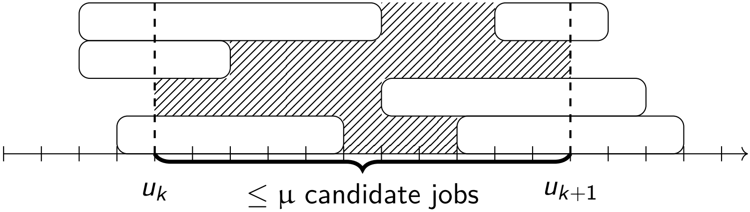

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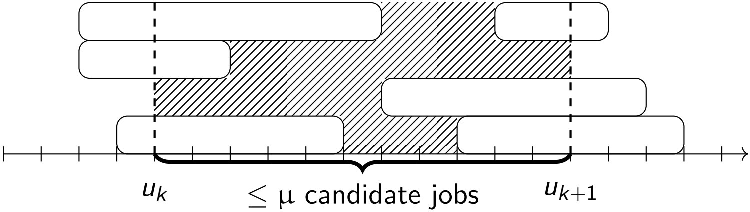

Parameter definition

Border flexibility $\beta = \max_{J_j \in \mathcal{J}}[\min(p_j - 1, \sigma_j + 1)]$.

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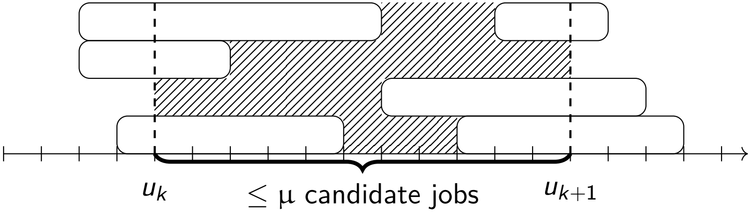

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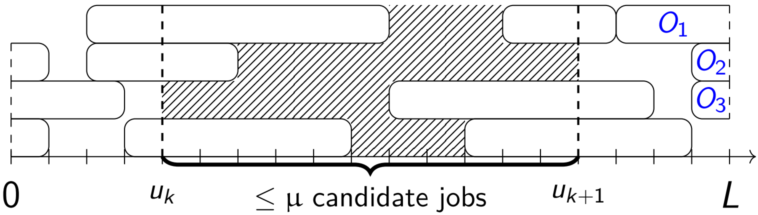
- $\mathcal{O}(2^{\mu} \cdot (\mu\beta + 1)^m)$ "border schedules" associated to u_k.
- Isolated days: in FPT$(\mu + \beta)$ [Hanen and Munier Kordon, 2023].

Border schedules



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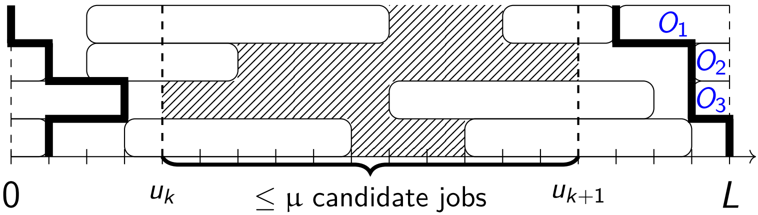

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Results: same as with isolated days

- $\mathcal{O}(\log(n))$-approximation algorithm.
- No $(2-\varepsilon)$-approximation algorithm (unless P $=$ NP).
- Fixed-parameter tractability: $(\mu+\beta)$ and $(m+\sigma)$.

Conclusion

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Future work

- Constant approximation algorithm?
- Improve the fixed-parameter algorithms.
  - $\mathcal{O}((\mu\beta+1)^{3m-1}\cdot 2^{3\mu}\cdot m^{\mu}\cdot\mu\cdot n+n\cdot\log(n))$ time.
- "Earliest Start Time" performance guarantee?
- Refine the model: unary/fixed L ; limited preemption.

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