Parameterized Complexity and New Efficient Enumerative Schemes for RCPSP

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[3]Université Paris Nanterre, F-92000 Nanterre, France,

18 November 2025

Outline

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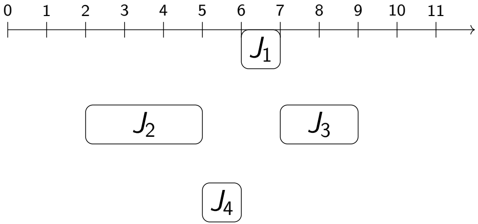
Scheduling



$\mathcal{J} = \{J_1, \ldots, J_n\}$ set of n jobs of processing time $p_1, \ldots, p_n$

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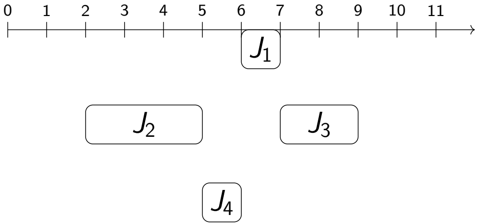
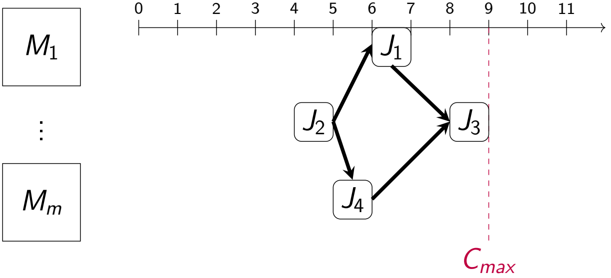


Notation: machines | job properties | optimization criterion

Scheduling



$$P|prec, p_j = 1|C_{max}$$

Resource-Constrained Project Scheduling Problem (RCPSP)

- INPUT:
  - a set $\mathcal{J}$ of n jobs J_j with processing time p_j,
  - a precedence graph *prec* on $\mathcal{J}$,
  - a set $\mathcal{R}$ renewable resources,
  - for each resource $\rho \in \mathcal{R}$ the available amount $\mathcal{R}_\rho$,
  - for each job $j \in \mathcal{J}$ the amount $res_{j,\rho} \leq \mathcal{R}_\rho$ of resource $\rho \in \mathcal{R}$ used during its execution.

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  - for every relation $i \to j$ in *prec* job i finishes before job j starts,
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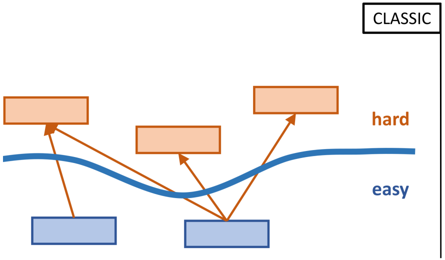
- $P|prec, p_j = 1|C_{max}$ is NP-complete [Ullman, 1975].

Motivation



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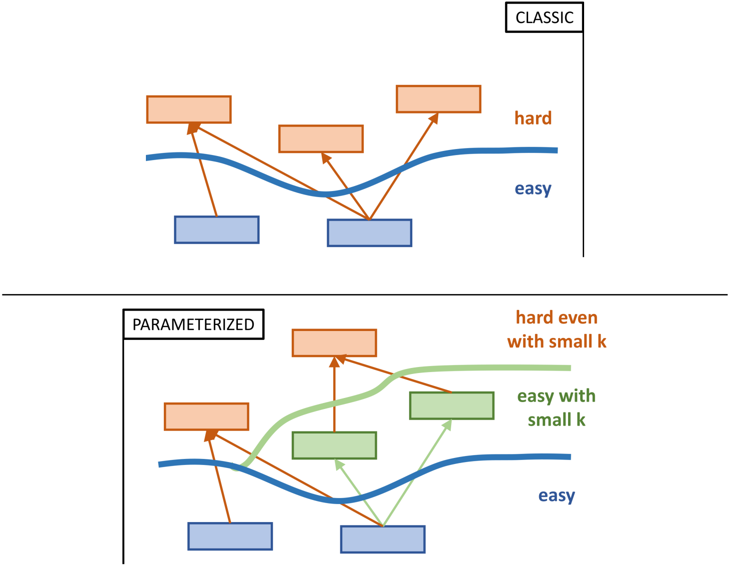
CLASSIC
hard
easy
PARAMETERIZED
hard even with small k
easy with small k
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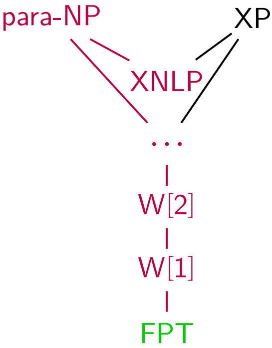
Going beyond NP-hardness

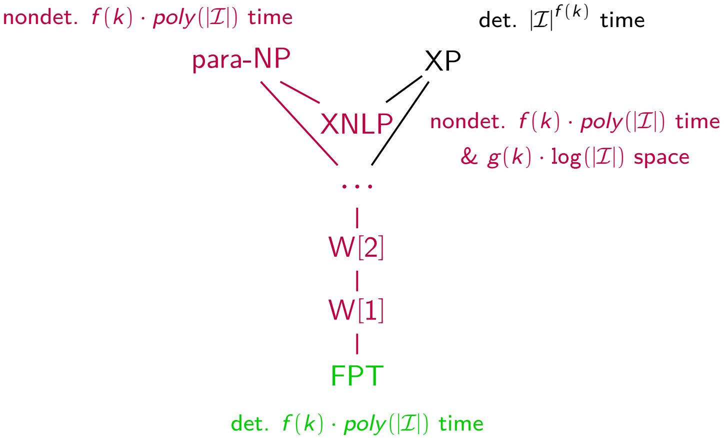
- Problem $\mathcal{P}$, instance $\mathcal{I}$.
- P: deterministic time $poly(|\mathcal{I}|)$.
- NP: nondeterministic time $poly(|\mathcal{I}|)$.

Going beyond NP-hardness

- Problem $\mathcal{P}$, instance $\mathcal{I}$.
  - Parameter $\bar{k}$, instance $\mathcal{I}$ with parameter value $\leq k$.
- P: deterministic time $poly(|\mathcal{I}|)$.
  - FPT: deterministic $f(k) \cdot poly(|\mathcal{I}|)$ time.
- NP: nondeterministic time $poly(|\mathcal{I}|)$.
  - para-NP: nondeterministic $f(k) \cdot poly(|\mathcal{I}|)$ time.


para-NP
XP
XNLP
...
W[2]
W[1]
FPT

Parameterized complexity classes



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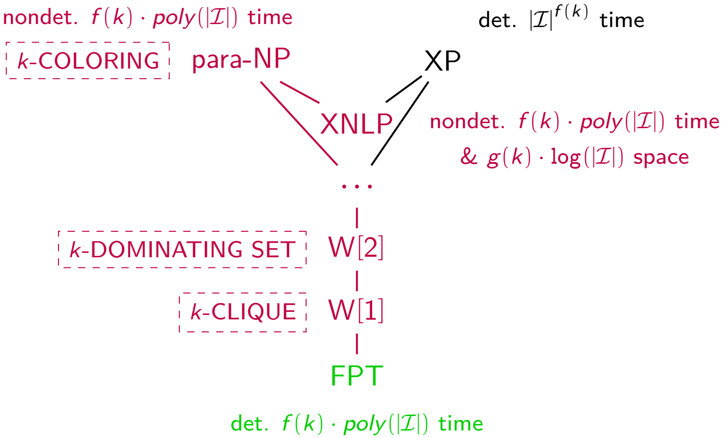

Example of parameterized results

- Parameters
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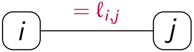
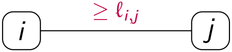
Objective

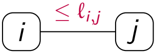
Apply this framework to other scheduling problems/parameters.

Outline

Precedence delays

- Given a feasible schedule and $(i \xrightarrow{\ell_{i,j}} j)$:
  - Exact delay: 

  - Minimum delay: 

  - Maximum delay: 

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- Parameter $\ell_{max} = max_{(i,j) \in prec}(\ell_{i,j})$.

Hardness with bounded precedence delays

- $1|chains(\ell^{ex}), \ell_{i,j} = \ell, p_j = 1|C_{max}$ parameterized by ℓ_{max} is W[1]-hard.

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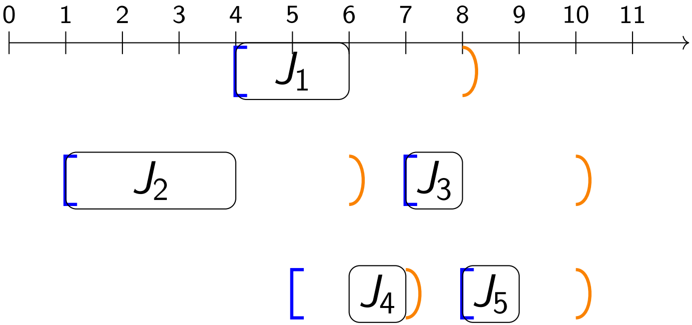
  - $\leftarrow$ $P|prec, p_j = 1|C_{max}$ parameterized by $m + w$ [Bodlaender et al., 2022].

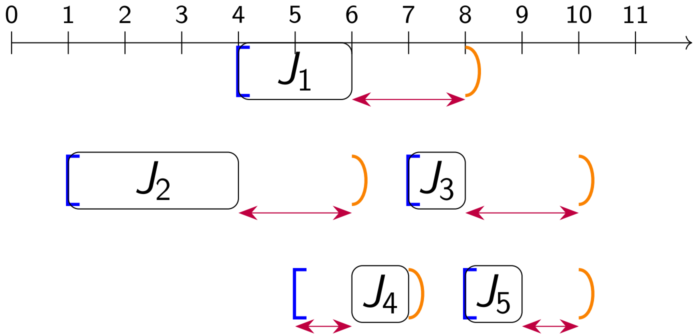
Outline

Time windows, slack σ and pathwidth pw



Release dates r_j, deadlines d_j.

Time windows, slack σ and pathwidth *pw*



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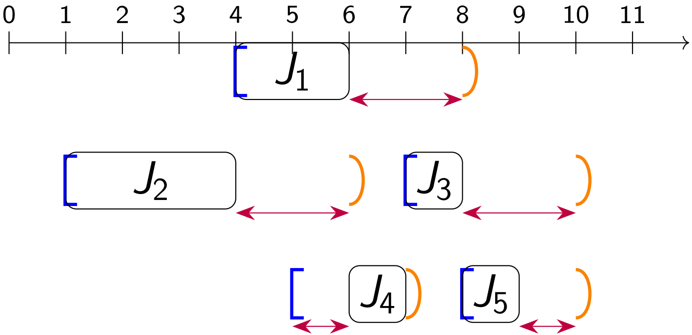
$\sigma = 2$



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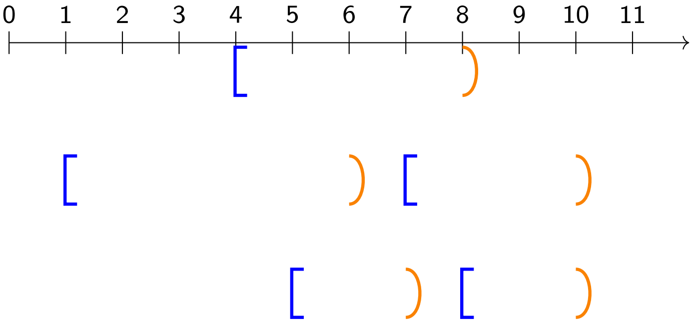

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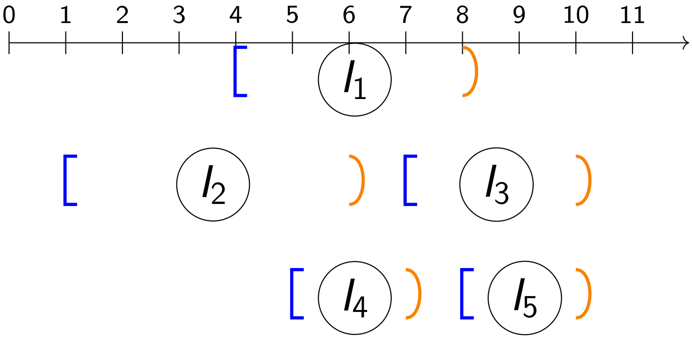

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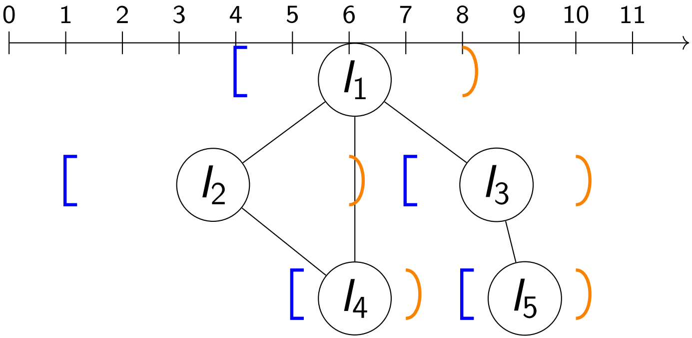

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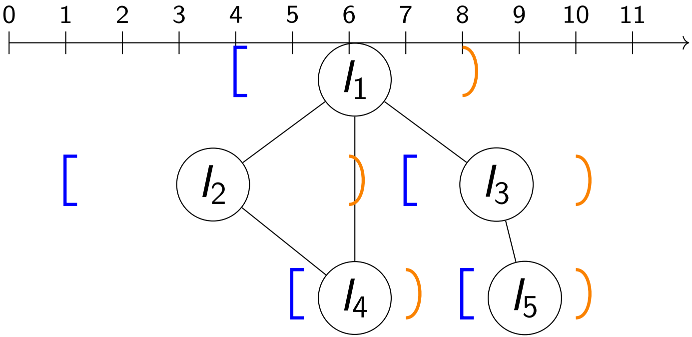
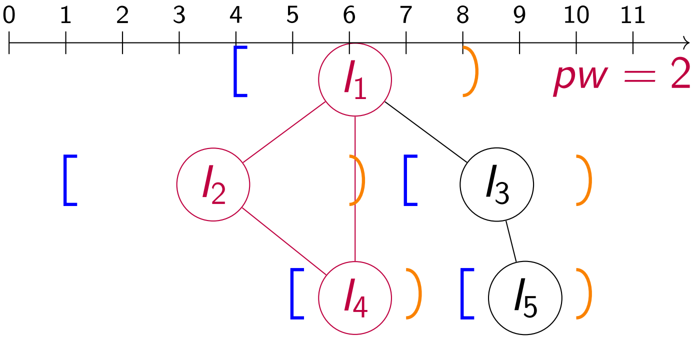


[Bodlaender and Möhring, 1993] For any interval graph G:

$$\omega(G) = \chi(G) = tw(G) + 1 = pw(G) + 1.$$

Time windows, slack σ and pathwidth *pw*



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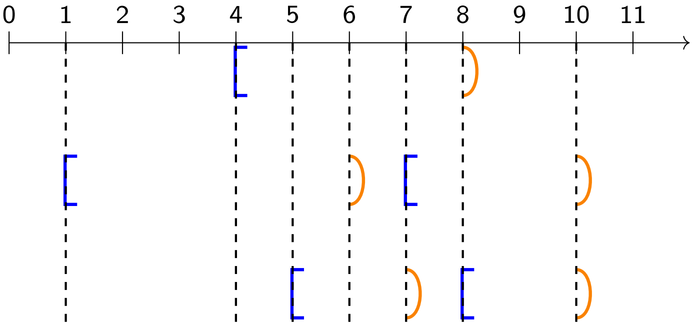

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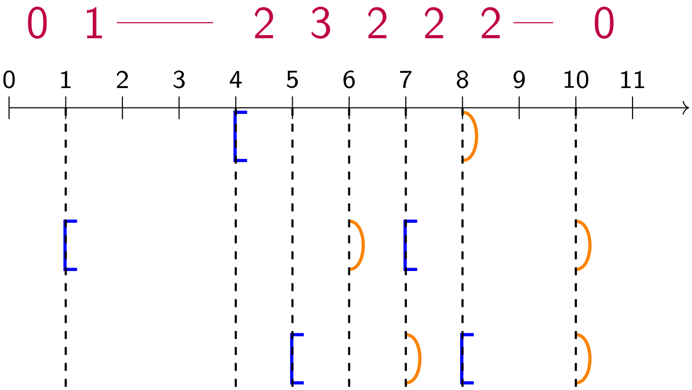

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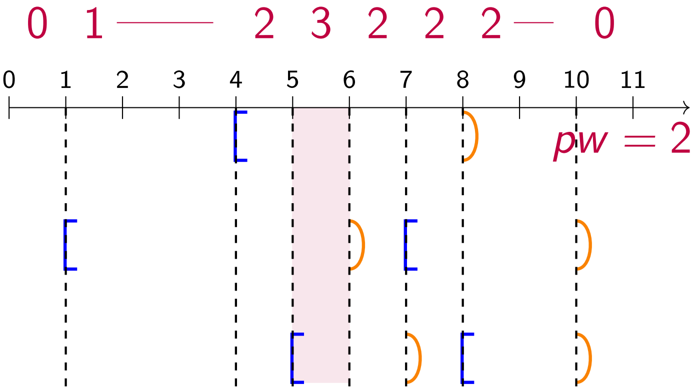

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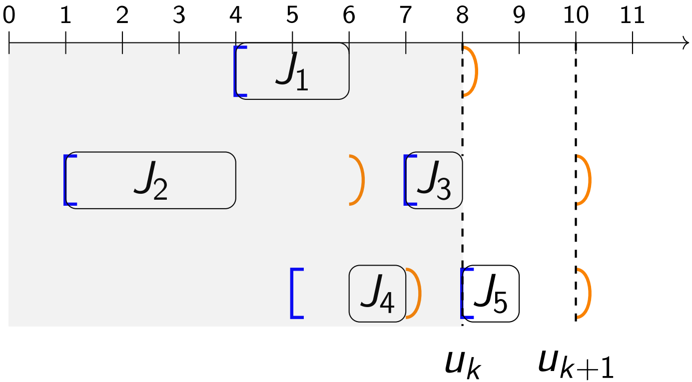

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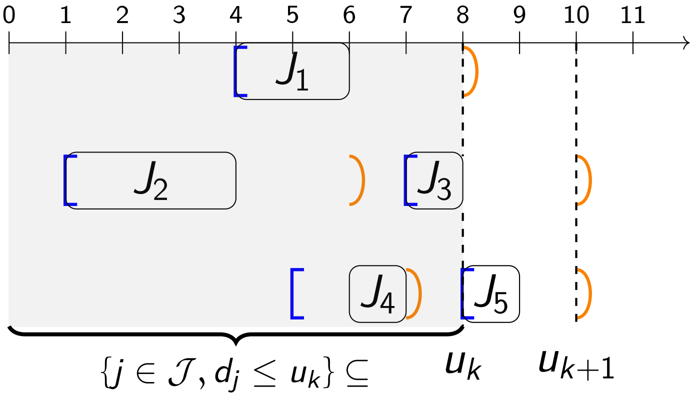

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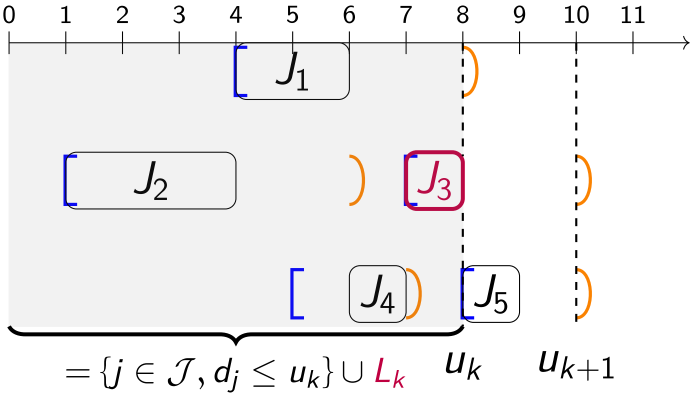

- $J_j \in L_k \implies [u_k, u_{k+1}) \subseteq [r_j, d_j)$: at most $pw + 1$ such jobs.

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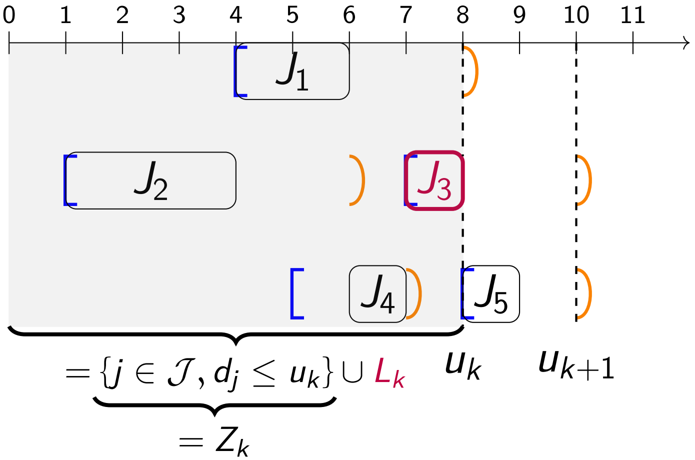

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Dynamic programming with pathwidth pw

Dynamic programming state (k, L_k)

- u_k = associated release date/deadline value.
- $Z_k \cup L_k$ = set of jobs starting before u_k.
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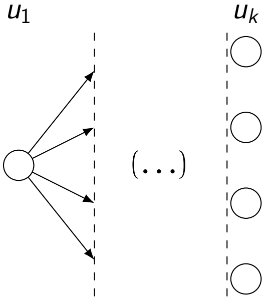

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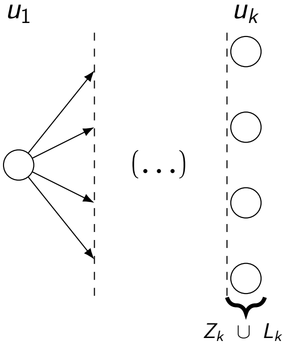

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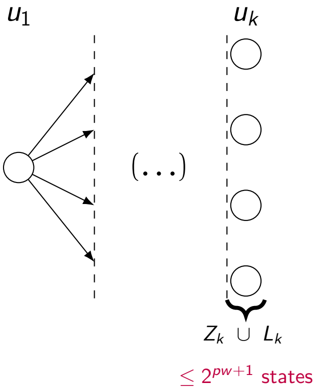

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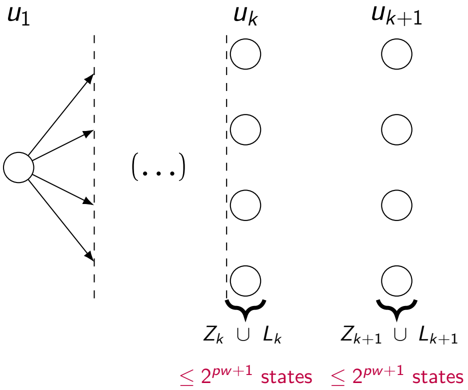

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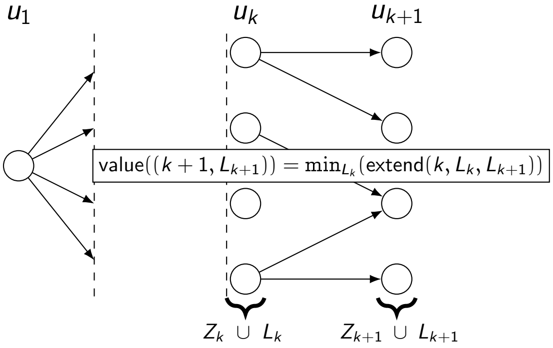

$\leq 2^{pw+1}$ states $\leq 2^{pw+1}$ states

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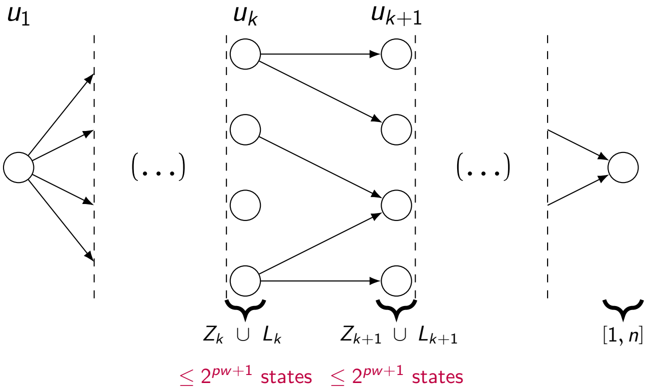

Some results with slack σ and pathwidth *pw*

- $1|prec, r_j, d_j|C_{max}$ param. by pw is in FPT.
  - $\mathcal{O}((pw+1)^2 4^{pw} n + n\log(n))$ time [Hanen et al., 2022].
- $P|prec, p_j = 1, r_j, d_j|C_{max}$ param. by pw is in FPT.
  - $\mathcal{O}(16^{pw} n^4)$ time [Munier Kordon, 2021].
  - $\mathcal{O}(pw 2^{pw} mn^3)$ time [Shi et al., 2026].

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- $P|\mathcal{M}_j(type), prec, r_j, d_j|C_{max}$ param. by $(pw+\sigma)$ is in FPT.
  - $\mathcal{O}([2\sigma(pw+1)]^{pw^2} n^4)$ time [Hanen and Munier Kordon, 2023]

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  - $\mathcal{O}(16^{pw} n^4)$ time [Munier Kordon, 2021].
  - $\mathcal{O}(pw 2^{pw} m n^3)$ time [Shi et al., 2026].
- $P|\mathcal{M}_j(type), prec, r_j, d_j|C_{max}$ param. by $(pw+\sigma)$ is in FPT.
  - $\mathcal{O}([2\sigma(pw+1)]^{pw^2} n^4)$ time [Hanen and Munier Kordon, 2023]

Question

- What about problems with precedence delays?

$1|chains(\ell^{ex}), p_j = 1, r_j, d_j|C_{max}$ with $pw \leq 2$ is NP-complete.

- Reduction from 3-COLORING: $G = (V, E)$.

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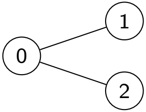

$\longrightarrow$

Scheduling instance with one machine, $3|V| + 2|E|$ jobs and exact delays.

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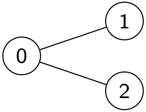

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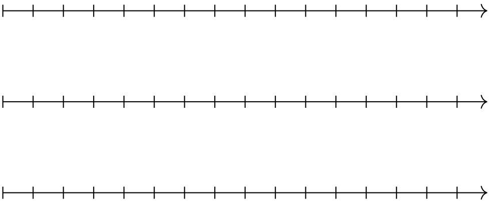
Vertex chain $\mathcal{C}_0$

Vertex chain $\mathcal{C}_1$

Vertex chain $\mathcal{C}_2$



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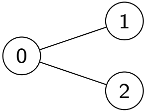

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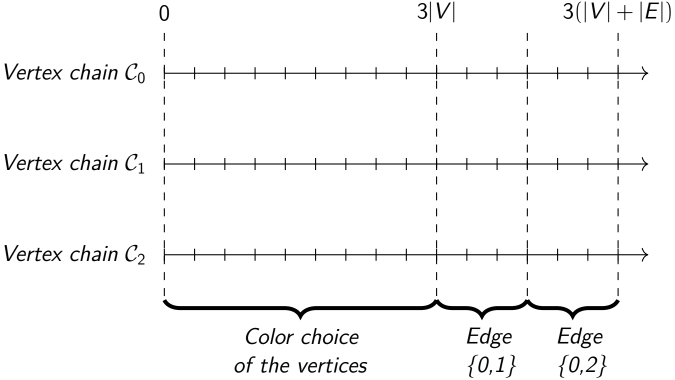

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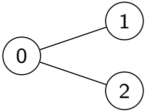

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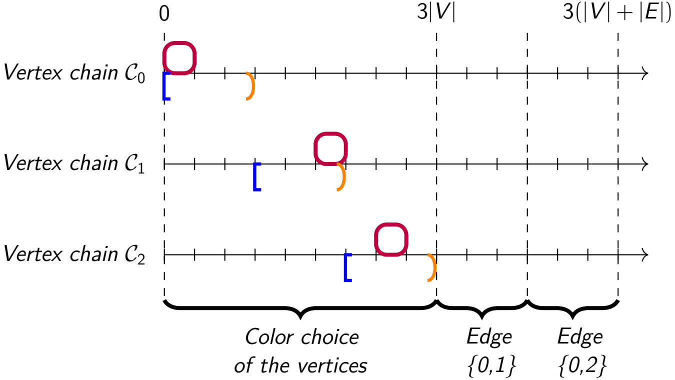

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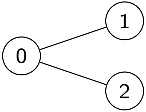

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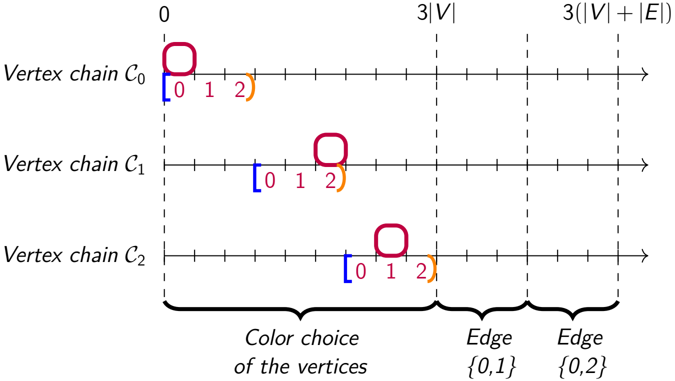

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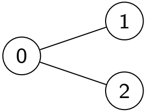

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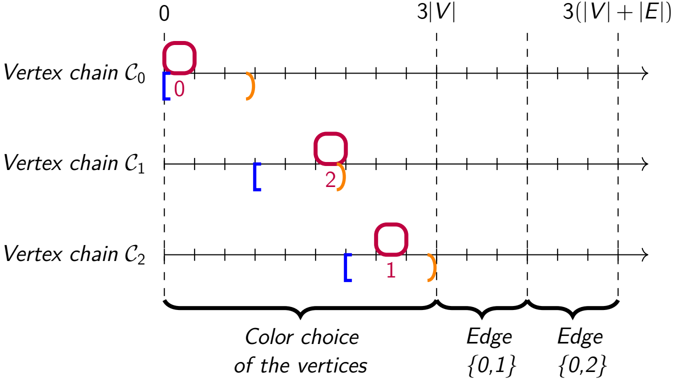

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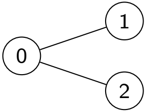

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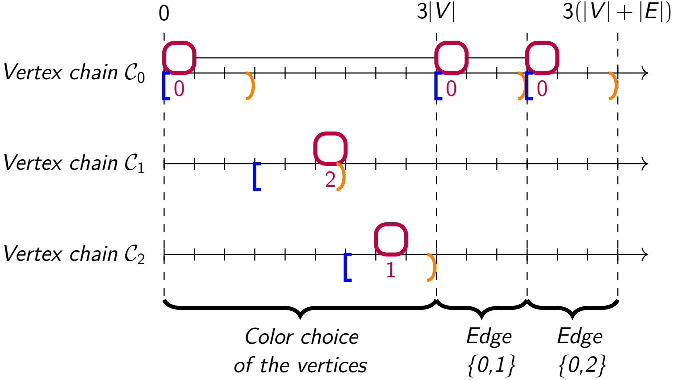

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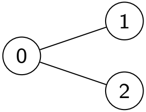

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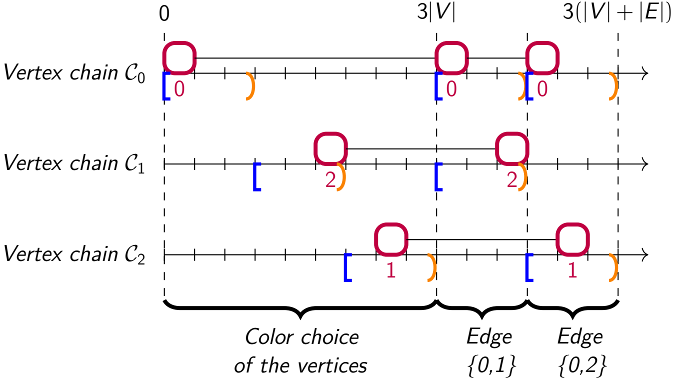

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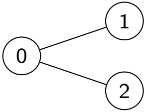

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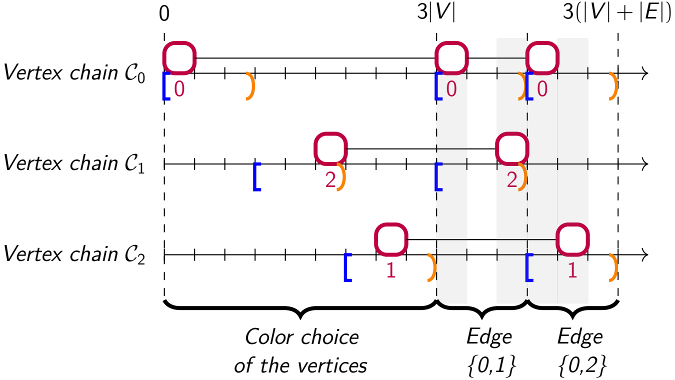

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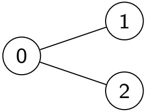

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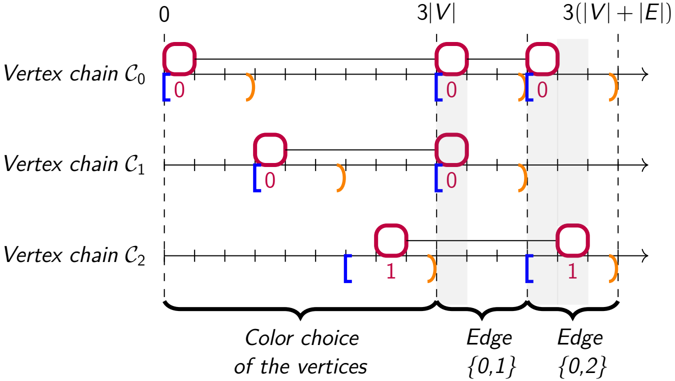

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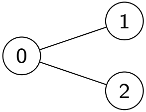

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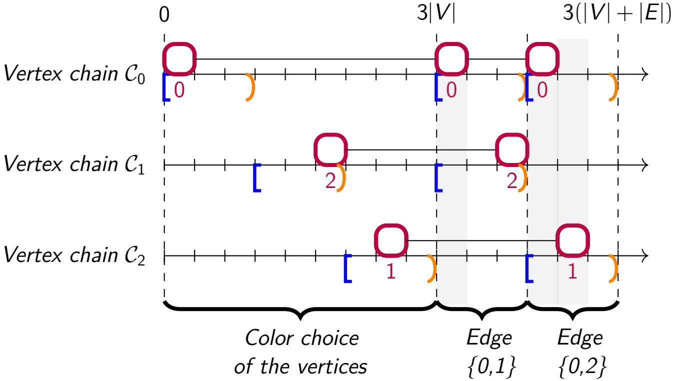
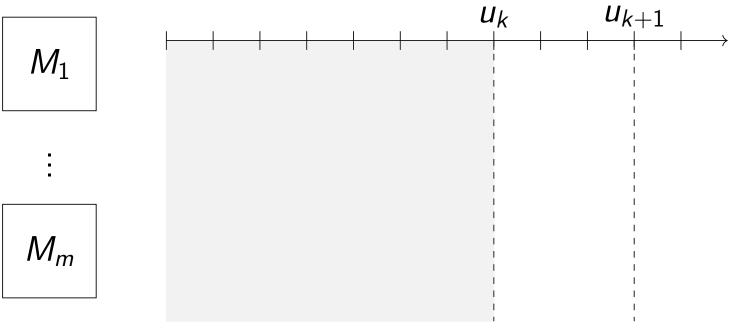


Outline

1. Introduction
2. Precedence delays
3. Adding time windows
4. **Pairing parameters**
5. Conclusion

$P|prec(\ell^{min}), p_j = 1, r_j, d_j|C_{max}$ is in FPT w.r.t. $(pw + \ell_{max})$.



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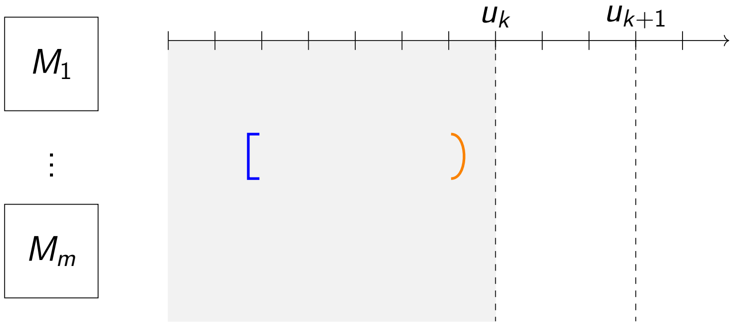

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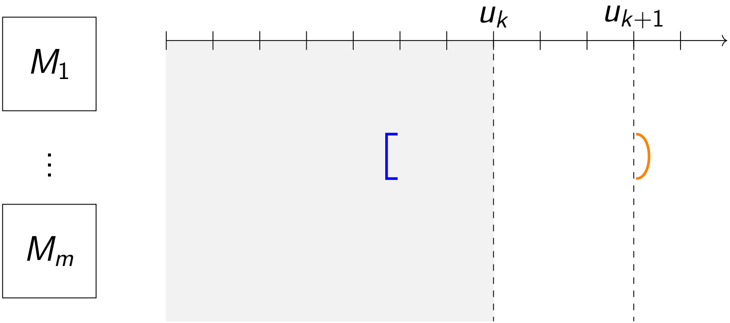

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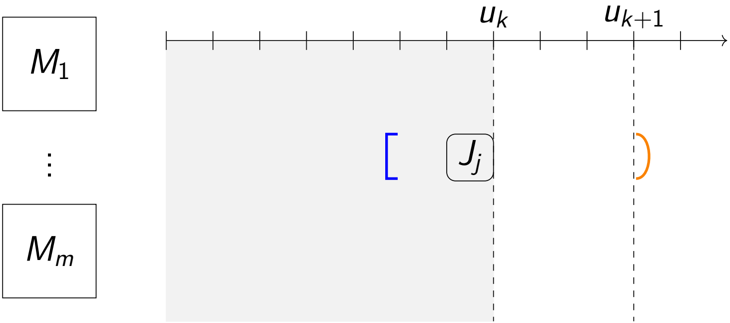

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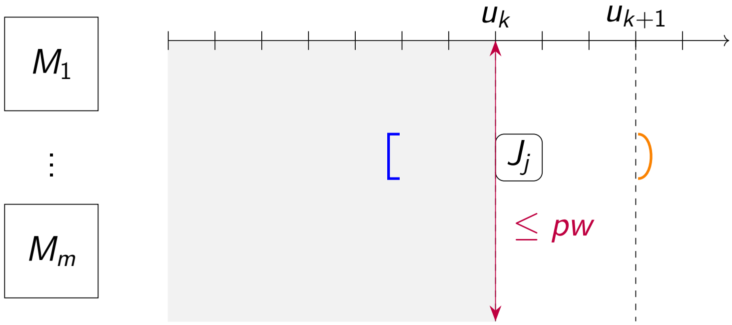

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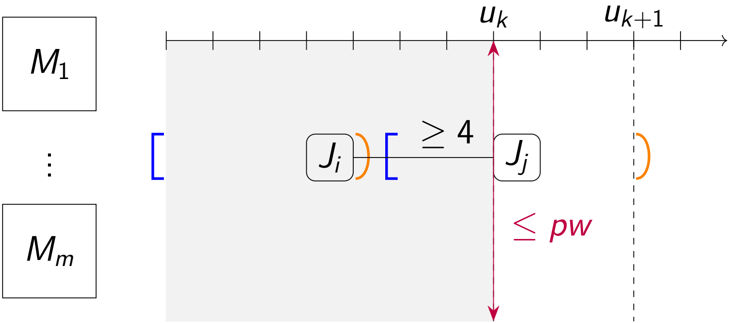

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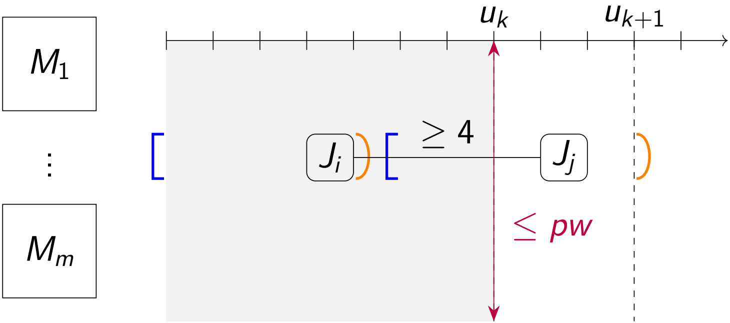

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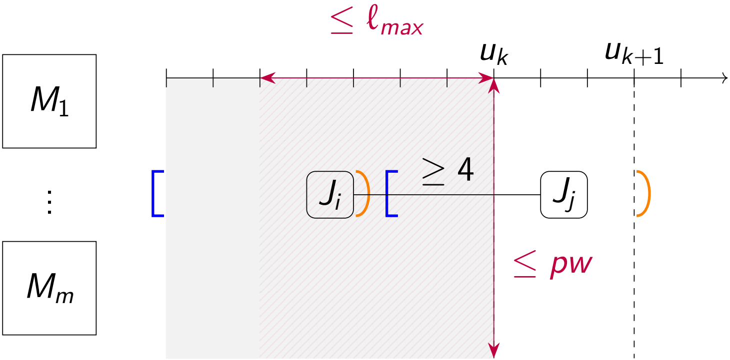

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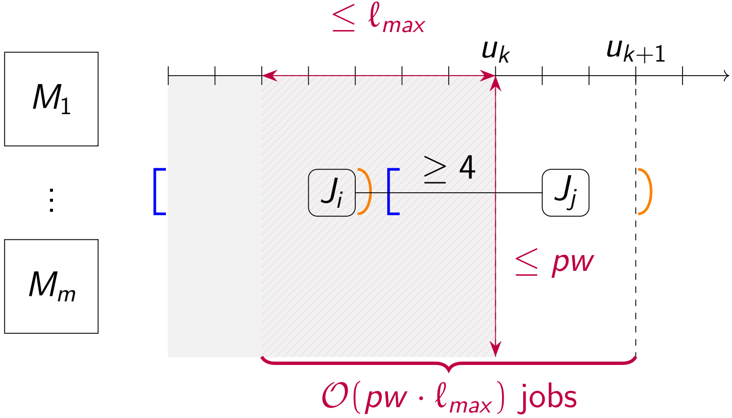

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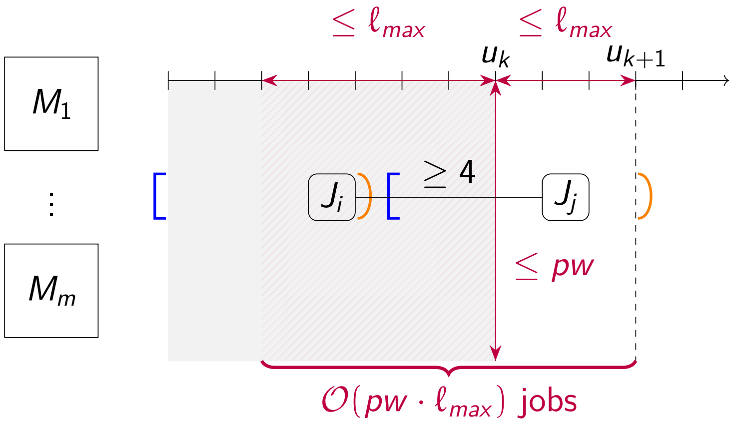

Outline

Conclusion

Parameter	$1\|prec(\ell^{min}), r_j, d_j\|C_{max}$	$P\|prec(\ell^{min}), p_j = 1, r_j, d_j\|C_{max}$
ℓ_{max}	XNLP-hard	
$pw + \sigma$	para-NP-complete	
$\sigma + \ell_{max}$	in FPT	para-NP-complete
$pw + \ell_{max}$	open	in FPT

Conclusion

Parameter	$1\|prec(\ell^{min}), r_j, d_j\|C_{max}$	$P\|prec(\ell^{min}), p_j = 1, r_j, d_j\|C_{max}$
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Future work

- Improve the existing algorithms.
    - Dependency on the parameter (fine-grained complexity).
    - Kernelization.

Conclusion

Parameter	$1 \vert prec(\ell^{min}), r_j, d_j \vert C_{max}$	$P \vert prec(\ell^{min}), p_j = 1, r_j, d_j \vert C_{max}$
ℓ_{max}	XNLP-hard	
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Future work

- Improve the existing algorithms.
  - Dependency on the parameter (fine-grained complexity).
  - Kernelization.
- Investigate other parameters/scheduling settings.
  - Postdoc: periodic time windows.

Postdoc: daily time windows

LIP, ENS de Lyon Nov. 2024 - Apr. 2026

- *Topic:* Scheduling radio-astronomical observations.

Postdoc: daily time windows

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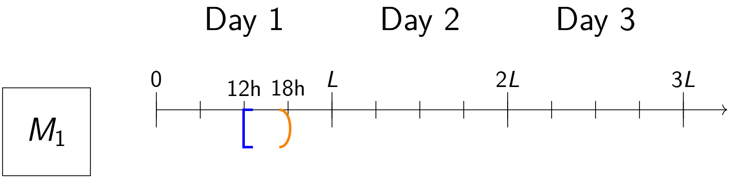

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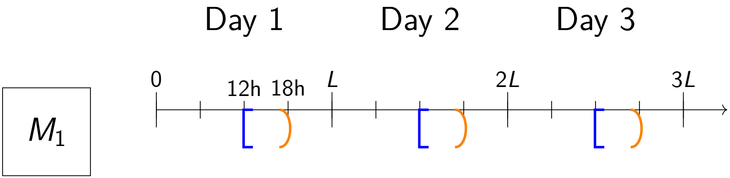

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