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Scheduling with release dates and deadlines: going beyond the pathwidth

Maher Mallem[1], Claire Hanen[2,3], Alix Munier-Kordon[2]

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[2]Sorbonne Université, CNRS, LIP6, F-75005 Paris, France,
[3]Université Paris Nanterre, F-92000 Nanterre, France,

6 November 2025

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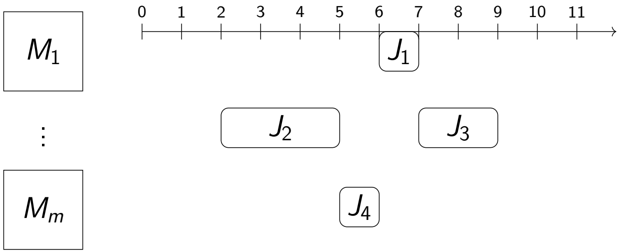
Scheduling



$\mathcal{J} = \{J_1, \ldots, J_n\}$ set of n jobs to schedule on m machines.

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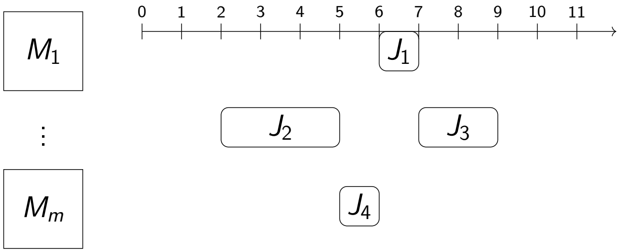

machines | job properties | optimization criterion

Scheduling

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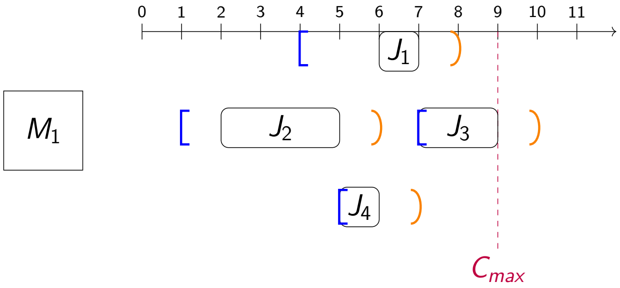

$$1|r_j, d_j|C_{max}$$

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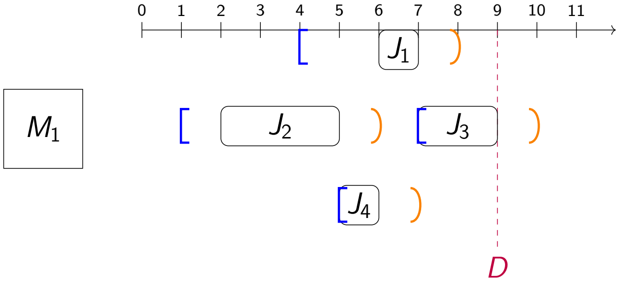
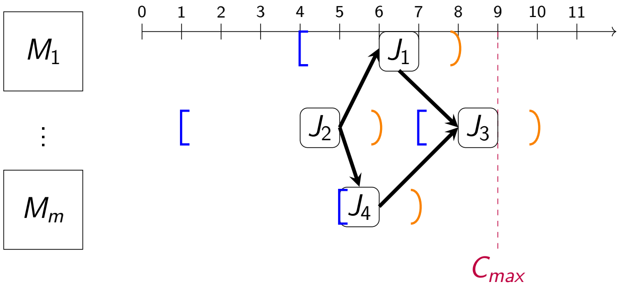


$$1|r_j, d_j|C_{max} \leq D$$

Scheduling



$$P|prec, p_j = 1, r_j, d_j|C_{max}$$

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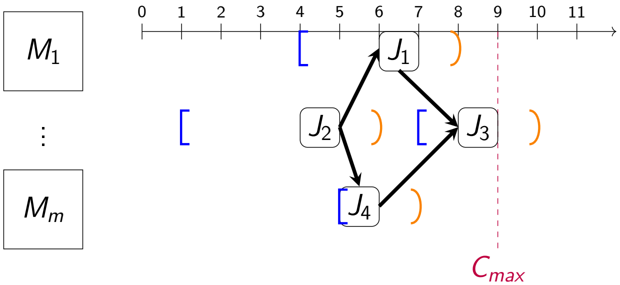

$1|r_j, d_j|C_{max}$ is NP-complete [Lenstra et al., 1977].

$P|prec, p_j = 1, r_j, d_j|C_{max}$ is NP-complete [Ullman, 1975].

Parameterized complexity classes

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Parameterized problem $(\mathcal{P}, \bar{k})$:

- parameter $\bar{k} : \mathcal{P} \mapsto \mathbb{Z}^+$,
- instance $\mathcal{I} \in \mathcal{P}$ such that $\bar{k}(\mathcal{I}) \leq k \in \mathbb{Z}^+$.

Parameterized complexity classes

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nondet. $f(k) \cdot poly(|\mathcal{I}|)$ time

det. $|\mathcal{I}|^{f(k)}$ time

para-NP

XP

$XNLP$

nondet. $f(k) \cdot poly(|\mathcal{I}|)$ time & $g(k) \cdot \log(|\mathcal{I}|)$ space

$\cdots$

$W[2]$

$W[1]$

FPT

det. $f(k) \cdot poly(|\mathcal{I}|)$ time

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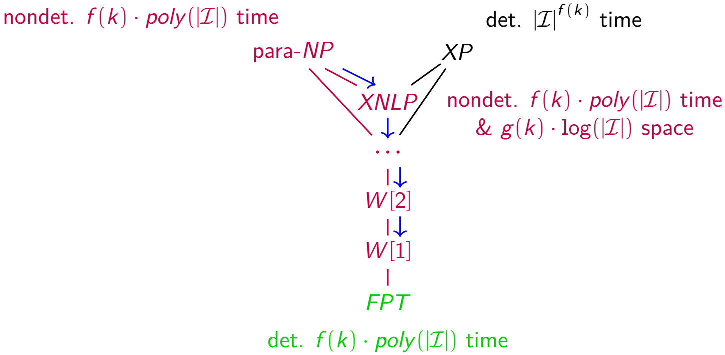

Pathwidth *pw*

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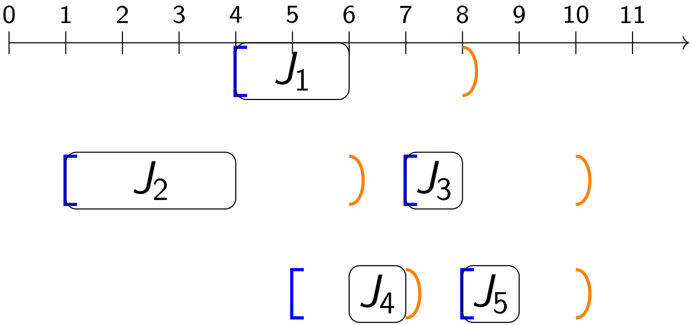

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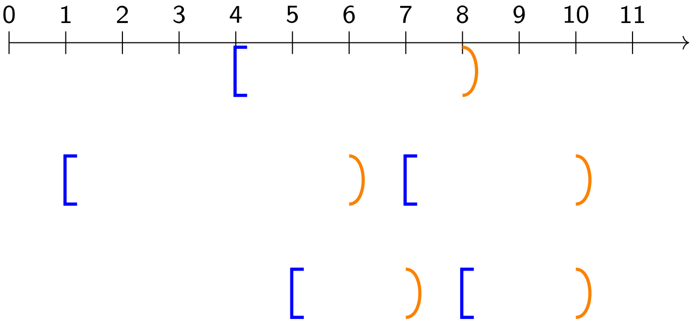

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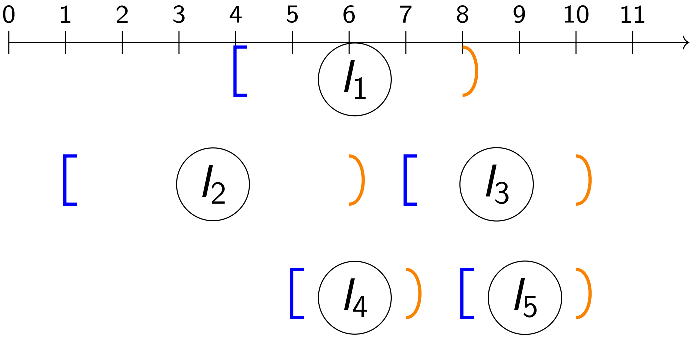

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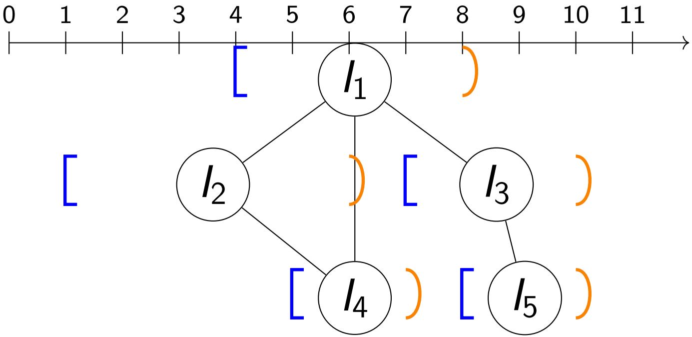

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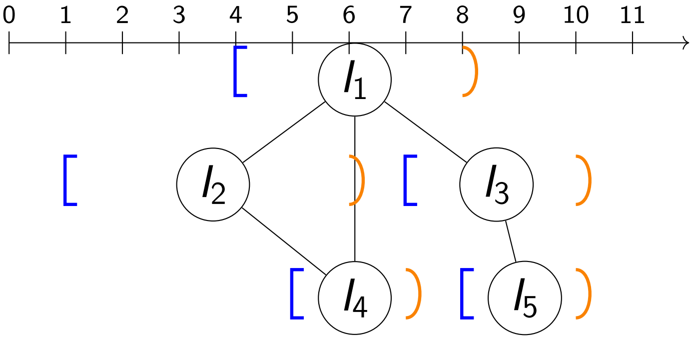

[Bodlaender and Möhring, 1993] For any interval graph G:

$$\omega(G) = \chi(G) = tw(G) + 1 = pw(G) + 1.$$

Pathwidth pw

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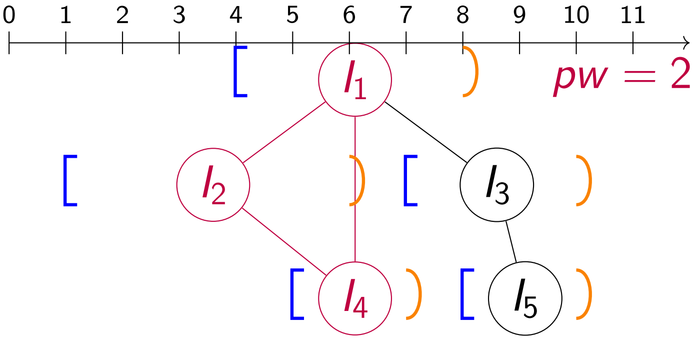

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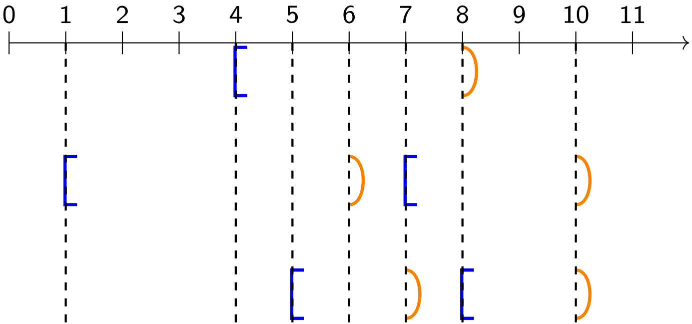

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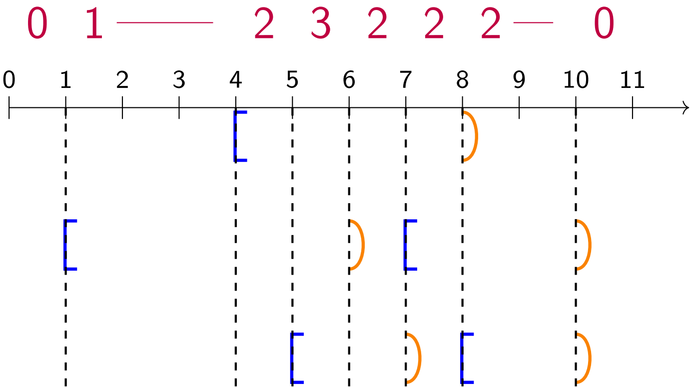

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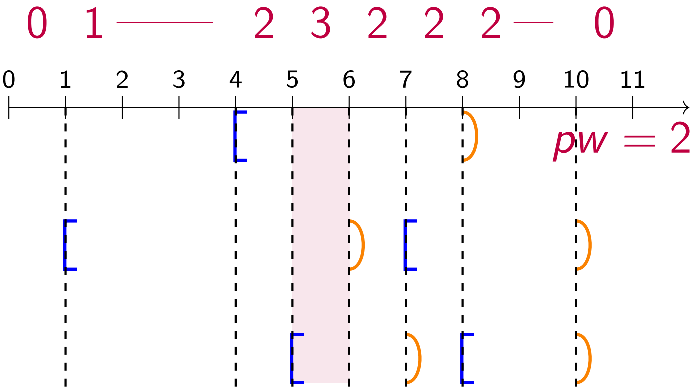

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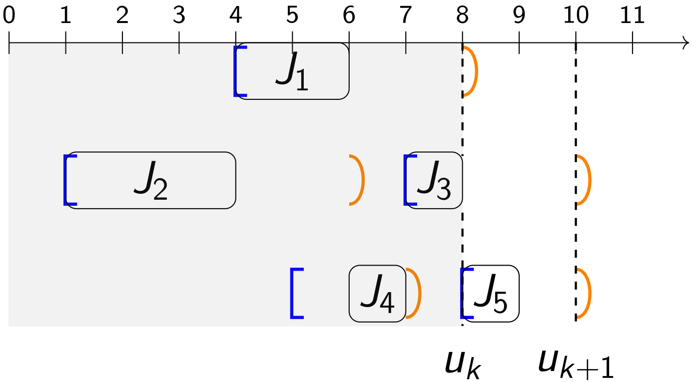

Pathwidth pw

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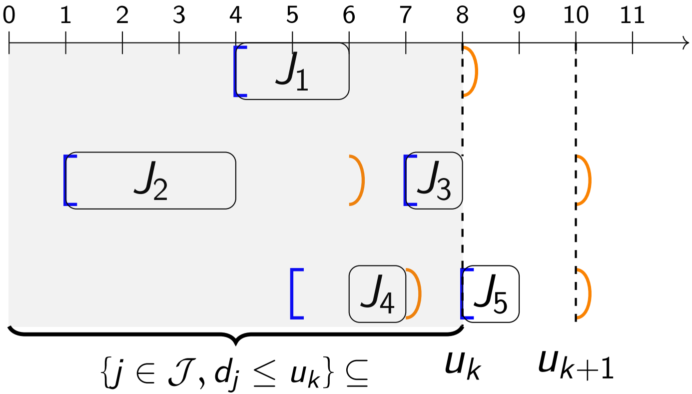

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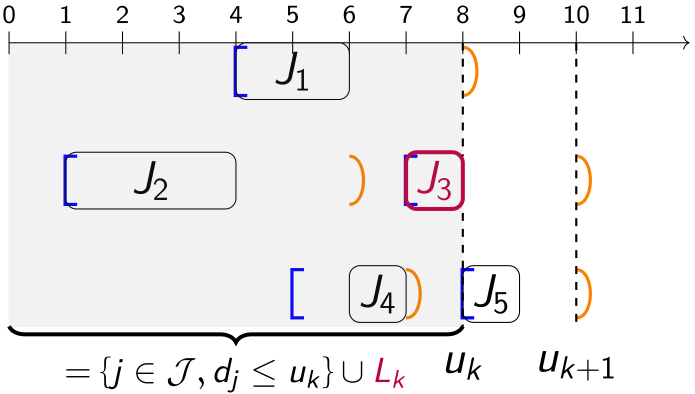
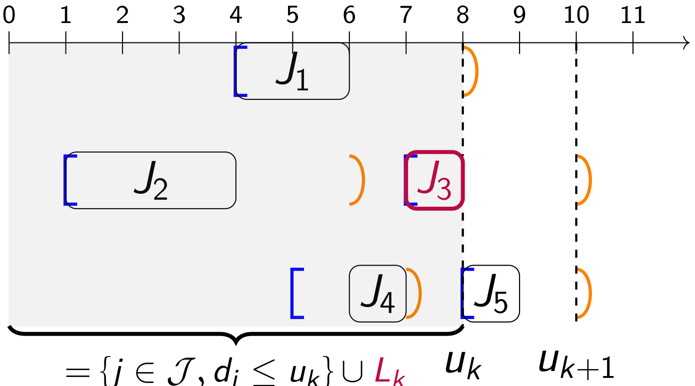


- $J_j \in L_k \implies [u_k, u_{k+1}) \subseteq [r_j, d_j)$: at most $pw + 1$ such jobs.

Pathwidth pw



$$= \underbrace{\{j \in \mathcal{J}, d_j \leq u_k\}}_{= Z_k} \cup L_k$$

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Dynamic programming with pathwidth pw

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Dynamic programming state (k, L_k)

- u_k = associated release date/deadline value.
- $Z_k \cup L_k$ = set of jobs starting before u_k.
- State value = minimum makespan among the associated partial schedules.

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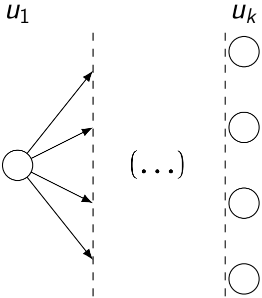

Dynamic programming with pathwidth *pw*

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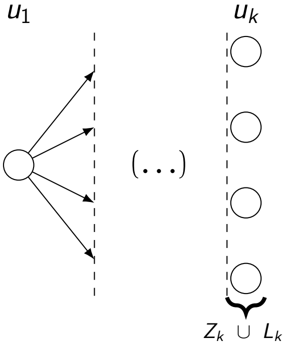

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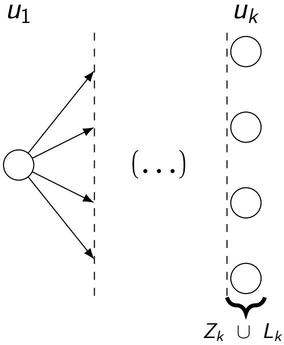

$\leq 2^{pw+1}$ states

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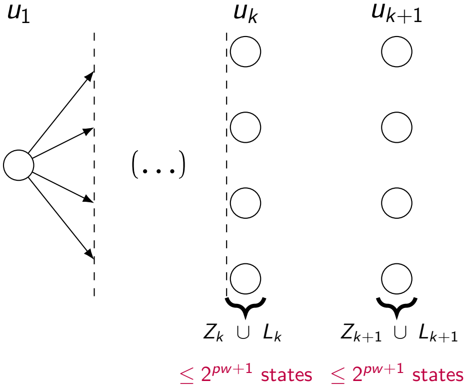

Dynamic programming with pathwidth pw

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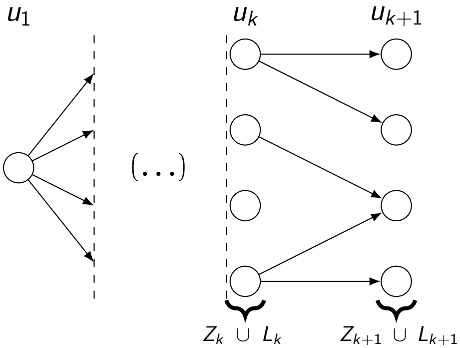

$\leq 2^{pw+1}$ states $\leq 2^{pw+1}$ states

Dynamic programming with pathwidth pw

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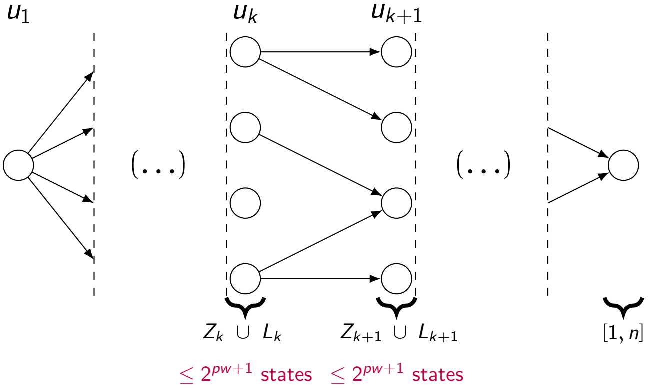

Recap (pathwidth pw)

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- $1|r_j, d_j|C_{max}$: in FPT.
  - [Hanen et al., 2022] $\mathcal{O}((pw+1)^2 4^{pw} \cdot n + n\log(n))$ time.
- $P|prec, p_j = 1, r_j, d_j|C_{max}$: in FPT.
  - [Munier Kordon, 2021] $\mathcal{O}(16^{pw} \cdot n^4)$ time.
  - [Shi et al., 2026] $\mathcal{O}(pw2^{pw} \cdot mn^3)$ time.
- $P2|r_j, d_j|C_{max}$: para-NP-complete [Hanen and Munier Kordon, 2023].

Recap (pathwidth pw)

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Questions

- FPT membership with smaller parameters?
- (Polynomial) kernelization?
- Other problems in FPT w.r.t. pw?

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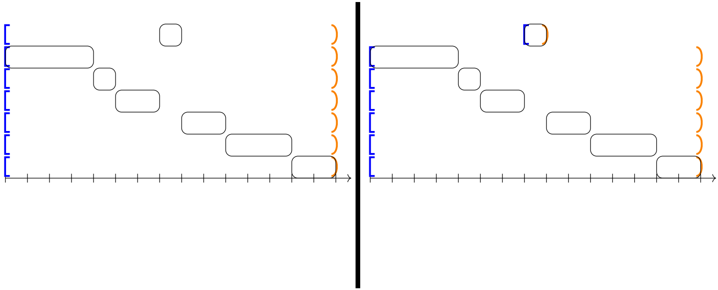
Proper level

- Two single machine instances with time windows.



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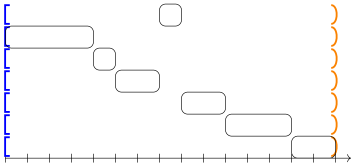
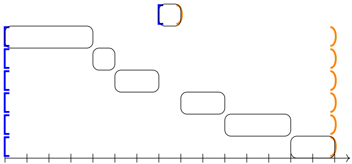


$pw = n - 1$

Interval graph: K_n



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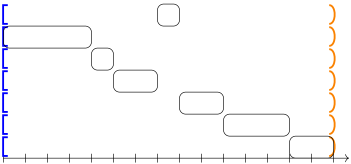

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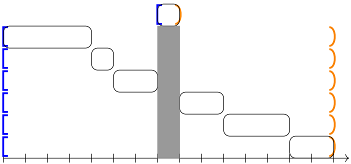

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Interval graph: K_n

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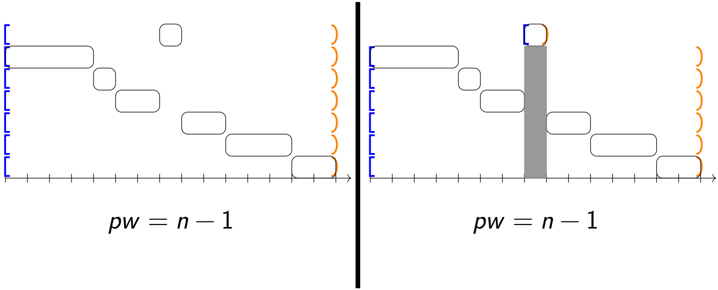

$pw = n - 1$ $\qquad$ $pw = n - 1$

Definition: Proper sets and proper level

- $\forall J_i \in \mathcal{J}, \Pi_i = \{J_j \in \mathcal{J}, r_j < r_i \wedge d_i < d_j\}$
- $pl = max_{J_i \in \mathcal{J}} \ |\Pi_i|$

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Proper level

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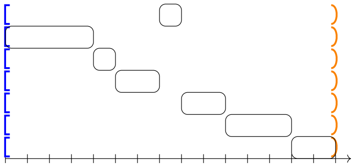
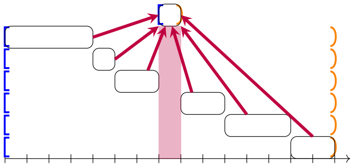


$pw = n - 1$ | $pl = 0$



$pw = n - 1$ | $pl = n - 1$

Definition: Proper sets and proper level

- $\forall J_i \in \mathcal{J}, \Pi_i = \{J_j \in \mathcal{J}, r_j < r_i \land d_i < d_j\}$
- $pl = max_{J_i \in \mathcal{J}} \ |\Pi_i|$

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Proper level

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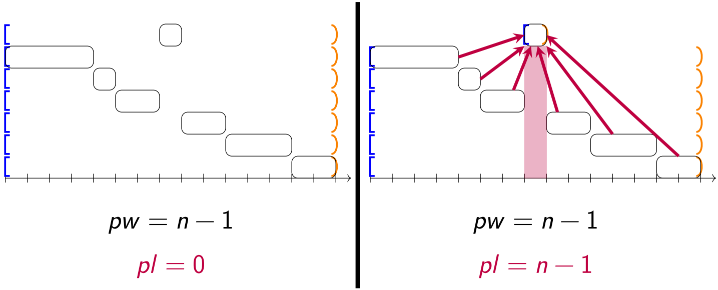

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The "Earliest Deadline first" rule

- "$\prec$" $=$ lexicographic order (nondecr. d_j, nondecr. r_j) on $\mathcal{J}$.
- Renaming the jobs of $\mathcal{J}$ in $[1, n]$ accordingly to $\prec$.

[Lawler and Moore, 1969] "Earliest Deadline first" rule (ED)

Schedule τ follows the (ED) rule if and only if:

$$\forall (i,j) \in \mathcal{J}^2, i \prec j \implies \tau(i) < \tau(j).$$

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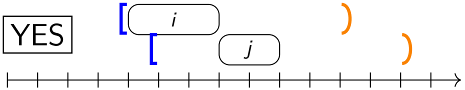

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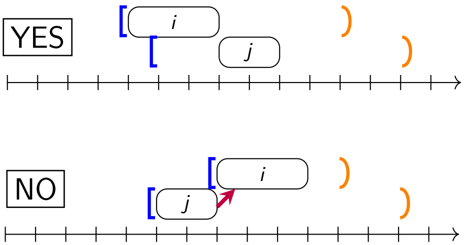

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Relaxing the "Earliest Deadline first" rule

Definition: "weak Earliest Deadline first" rule (wED)

Schedule τ follows the (wED) rule if and only if:
$\forall (i,j) \in \mathcal{J}^2, i \prec j \implies$

(1) $\tau(i) < \tau(j)$, or

(2) $j \in \Pi_i$, or

(3) $\exists h \prec i, \tau(j) < \tau(h) < \tau(i)$.

Relaxing the “Earliest Deadline first” rule

Definition: “weak Earliest Deadline first” rule (wED)

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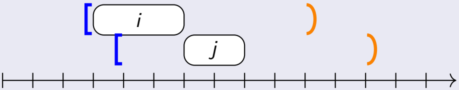
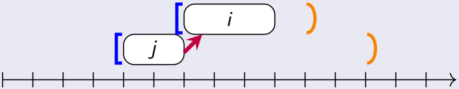


(2) $j \in \Pi_i$, or



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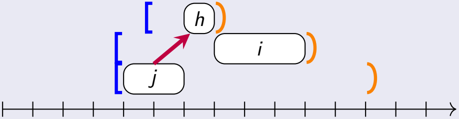

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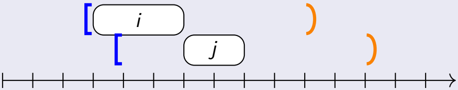

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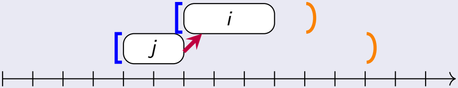

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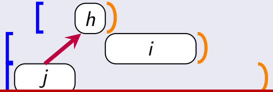

Proposition

On $1|r_j, d_j|C_{max}$ the (wED)-following schedules are dominant.

Summits

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Definition: Summit

A summit is a job i in $\mathcal{J}$ such that: $\forall j \in \mathcal{J}, i \notin \Pi_j$.

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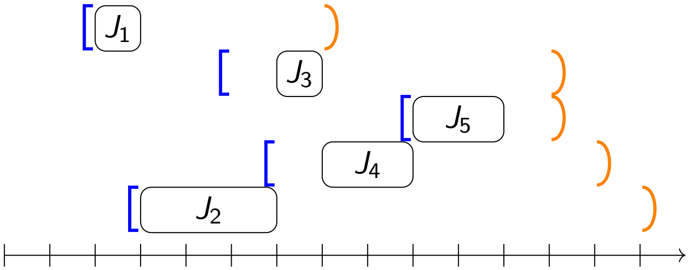

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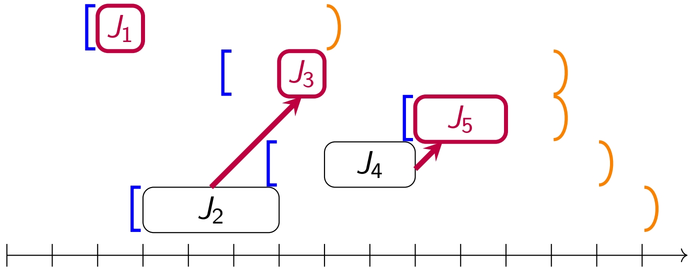

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Summit-based schedule enumeration

Lemma

Let $s_1 \prec \ldots \prec s_N$ be the summits of the instance ($N \leq n$).
If a schedule τ follows the (wED) rule, then:

(i) τ is of the form $T_1 s_1 \ldots s_k T_{k+1} s_{k+1} \ldots s_N T_{N+1}$,

(ii) if $i \prec s_k$ then $\tau(i) < \tau(s_k)$,

(iii) if $s_k \prec j$ and $\tau(j) < \tau(s_k)$ then $j \in \Pi_{s_k}$.

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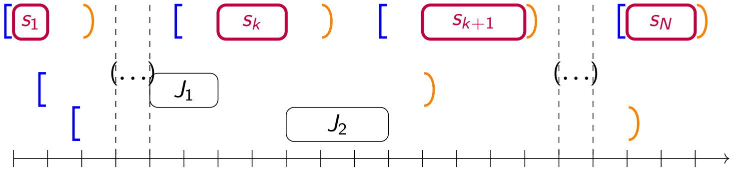

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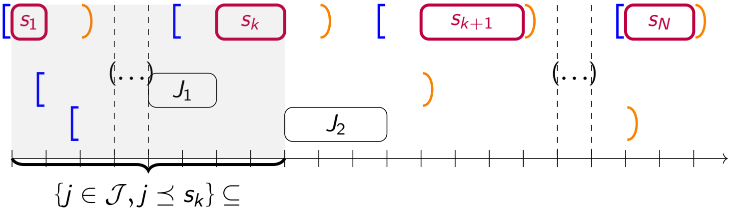

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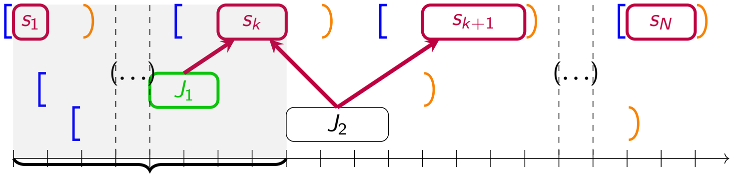

$= \{j \in \mathcal{J}, j \preceq s_k\} \cup L_k$

- $j \in L_k \implies j \in \Pi_{s_k}$: at most pl such jobs j.

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Recap (smaller parameters)

- Smaller parameters than pw in the interval graph.
  (clique-width, twin-width, proper thinness etc.)
  - $1|r_j, d_j|C_{max}$: para-NP-complete.
- Proper level pl.
  - $1|r_j, d_j|C_{max}$: in FPT ($\mathcal{O}((pl+1)^2 4^{pl} \cdot N + n\log(n))$ time).

Recap (smaller parameters)

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Recap (smaller parameters)

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Open problems

- $1|r_j, d_j|C_{max}$: smaller parameter than pl?
- $P|prec, p_j = 1, r_j, d_j|C_{max}$: smaller parameter than pw?

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Kernelization

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- Decision problem $\mathcal{P}$.
  - Parameter $\bar{k}$, instance $\mathcal{I}$ with parameter value $\leq k$.
- P: deterministic $poly(|\mathcal{I}|)$ time.
  - FPT: deterministic $f(k) \cdot poly(|\mathcal{I}|)$ time.

Kernelization

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- Kernelization algorithm: outputs $\mathcal{I}'$ in $poly(|\mathcal{I}|)$ time such that:
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- Polynomial kernel: f is polynomial.

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No polynomial kernel w.r.t. pathwidth pw

Proposition

Unless $NP \subseteq coNP/poly$ there is no polynomial kernel of $1|r_j, d_j|\star$ w.r.t. pathwidth pw.

No polynomial kernel w.r.t. pathwidth pw

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Unless $NP \subseteq coNP/poly$ there is no polynomial kernel of $1|r_j, d_j|\star$ w.r.t. pathwidth pw.

- *Proof:* Cross-composition from PARTITION.
  - $(P_1, \ldots, P_t) \mapsto \mathcal{I}$,
  - $pw \leq poly(max_i(|P_i|) + log(t))$.

No polynomial kernel w.r.t. pathwidth pw

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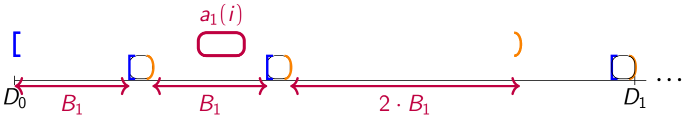

No polynomial kernel w.r.t. pathwidth *pw*

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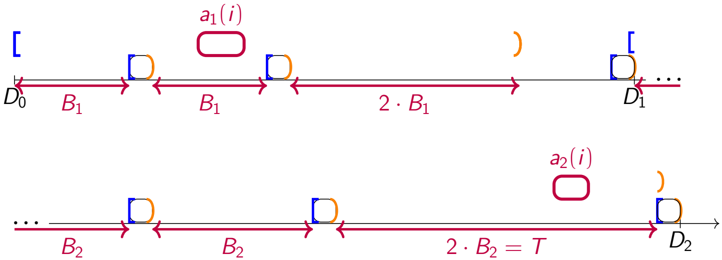

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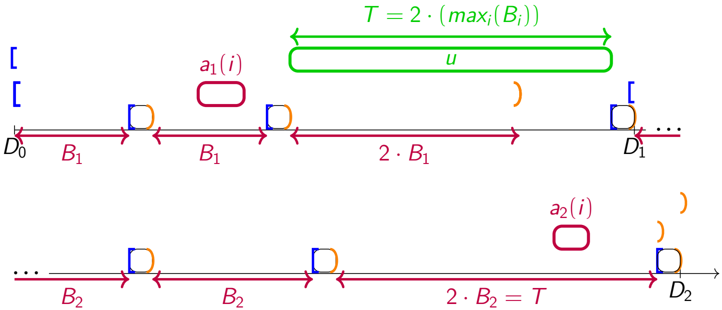

Pathwidth pw and vertex cover number vc

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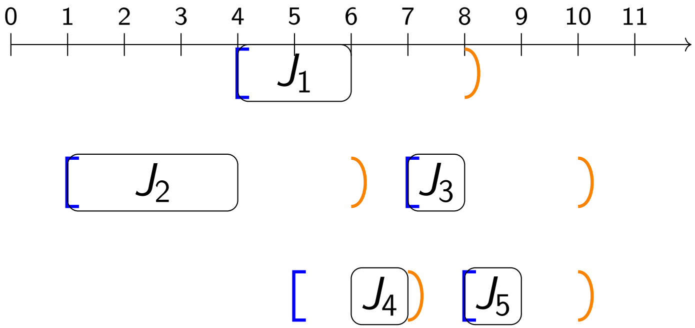

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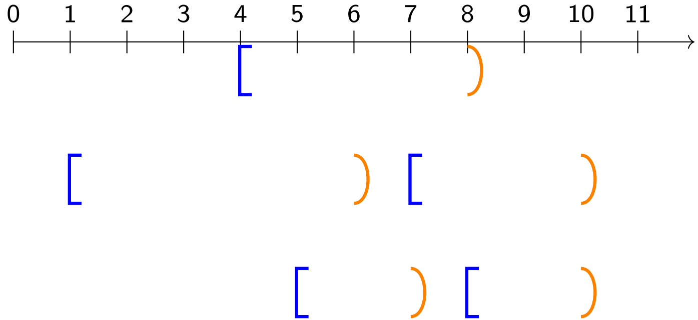

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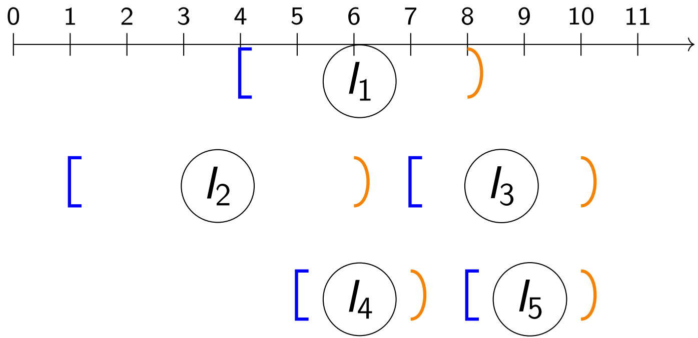

Pathwidth *pw* and vertex cover number *vc*

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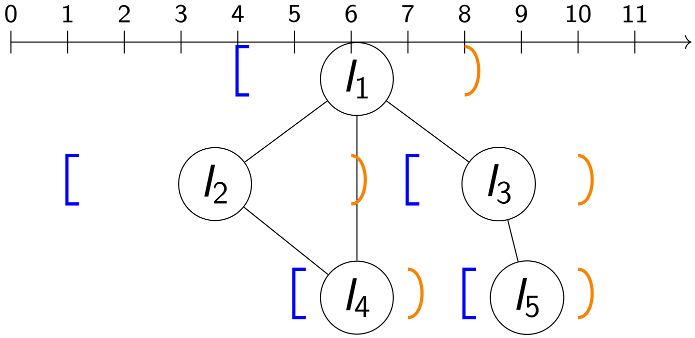

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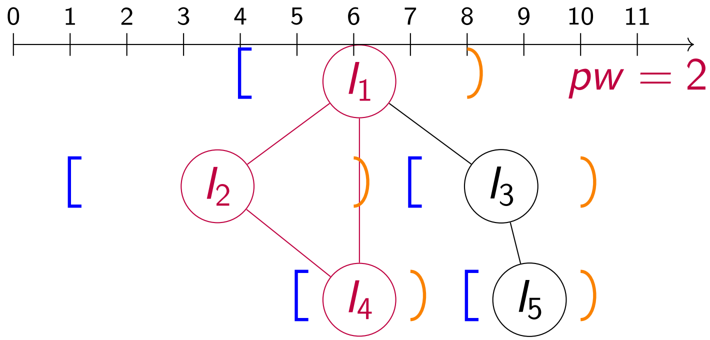

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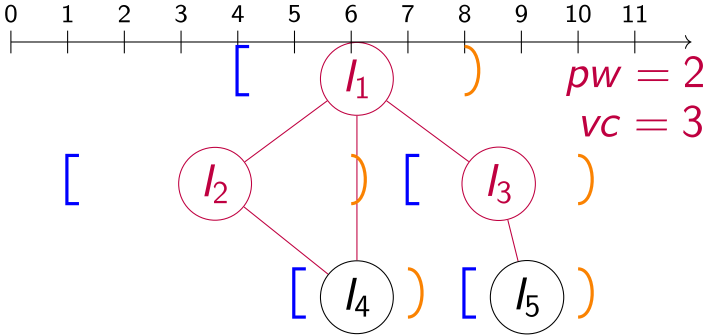

$pw = 2$

$vc = 3$

Pathwidth pw and vertex cover number vc

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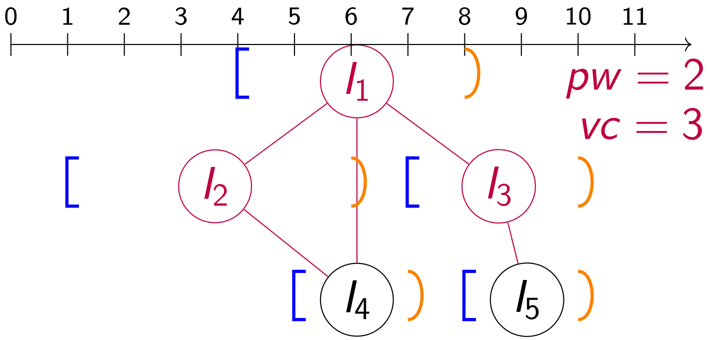

Claim

$$pw \leq vc$$

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Polynomial kernel overview

Proposition

$1|r_j, d_j|\star$ has a polynomial kernel w.r.t. parameter vc.

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Proposition

$1|r_j, d_j|\star$ has a polynomial kernel w.r.t. parameter vc.

$$\boxed{\begin{array}{c}\text{Instance } \mathcal{I} \\ n \text{ jobs} \\ \text{size } \mathcal{O}(n \cdot log(max_j(d_j)))\end{array}} \xrightarrow{(1)} \boxed{\begin{array}{c}\text{Instance } \mathcal{I}' \\ \mathcal{O}(vc^2) \text{ jobs} \\ \text{size } \mathcal{O}(vc^2 \cdot log(max_j(d_j)))\end{array}} \xrightarrow{(2)} \boxed{\begin{array}{c}\text{Instance } \mathcal{I}'' \\ \mathcal{O}(vc^2) \text{ jobs} \\ \text{size } \mathcal{O}(vc^8)\end{array}}$$

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Proposition

$1|r_j, d_j|\star$ has a polynomial kernel w.r.t. parameter vc.

Instance $\mathcal{I}$		Instance $\mathcal{I}'$		Instance $\mathcal{I}''$
n jobs size $\mathcal{O}(n \cdot log(max_j(d_j)))$	$\xrightarrow{(1)}$	$\mathcal{O}(vc^2)$ jobs size $\mathcal{O}(vc^2 \cdot log(max_j(d_j)))$	$\xrightarrow{(2)}$	$\mathcal{O}(vc^2)$ jobs size $\mathcal{O}(vc^8)$

- (1) Delete *most* jobs.
  - Use a reduction rule.
  - Time $\mathcal{O}(n^2)$.
- (2) Reduce time values.
  - Translate into a binary IP and use a theorem from [Frank and Tardos, 1987].
  - Time $\mathcal{O}(n^{18})$.

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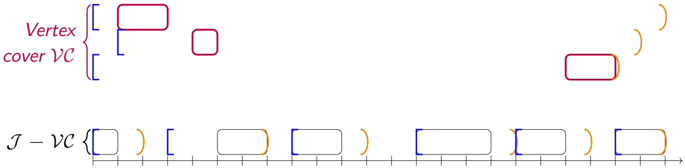
(1) Viewing non-vertex covering jobs as gaps



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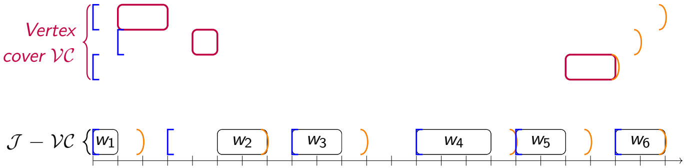
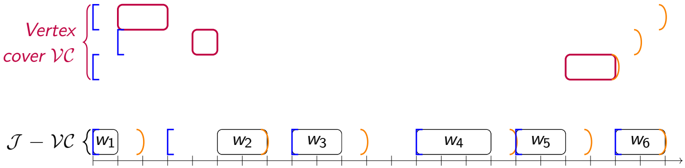


- $\mathcal{J} - \mathcal{VC}$: jobs $w_1, w_2, \ldots, w_{|\mathcal{J} - \mathcal{VC}|}$.

(1) Viewing non-vertex covering jobs as gaps



- $\mathcal{J} - \mathcal{VC}$: jobs $w_1, w_2, \ldots, w_{|\mathcal{J}-\mathcal{VC}|}$.
- Gap = two consecutive jobs w_i, w_{i+1}.

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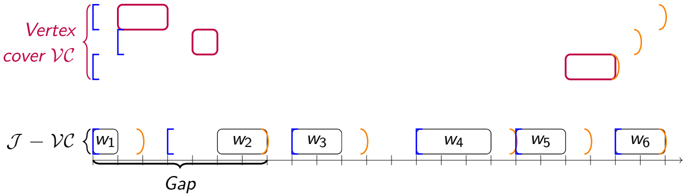

- $\mathcal{J} - \mathcal{VC}$: jobs $w_1, w_2, \ldots, w_{|\mathcal{J}-\mathcal{VC}|}$.
- Gap $=$ two consecutive jobs w_i, w_{i+1}.

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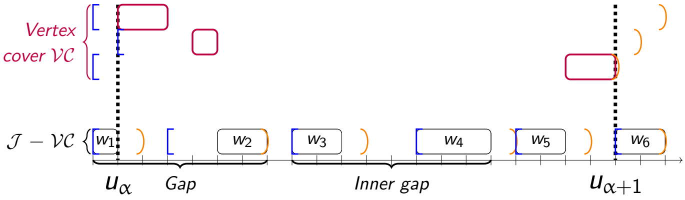

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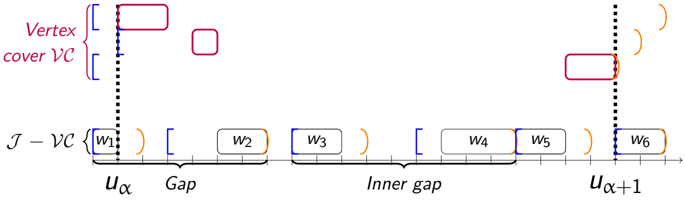

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- Maximum capacity: w_i at the earliest, w_{i+1} at the latest.

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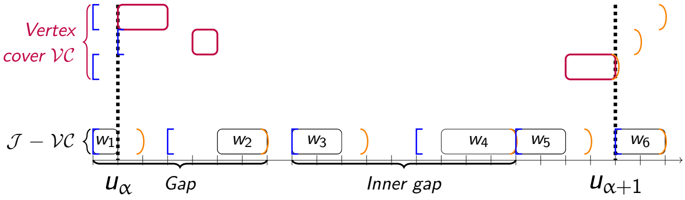

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  - Inner gap: time windows fully included in $[u_\alpha, u_{\alpha+1})$.
- Maximum capacity: w_i at the earliest, w_{i+1} at the latest.
- $vc_\alpha = |\{J_j \in \mathcal{VC}, [u_\alpha, u_{\alpha+1}) \subseteq [r_j, d_j)\}|$.

Reduction rule

In each interval $[u_\alpha, u_{\alpha+1})$, only keep the $3vc_\alpha$ inner gaps with highest maximum capacity.

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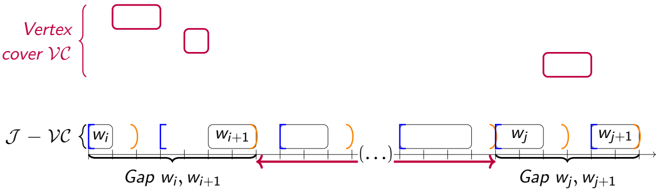
(1) Deleting unnecessary inner gaps



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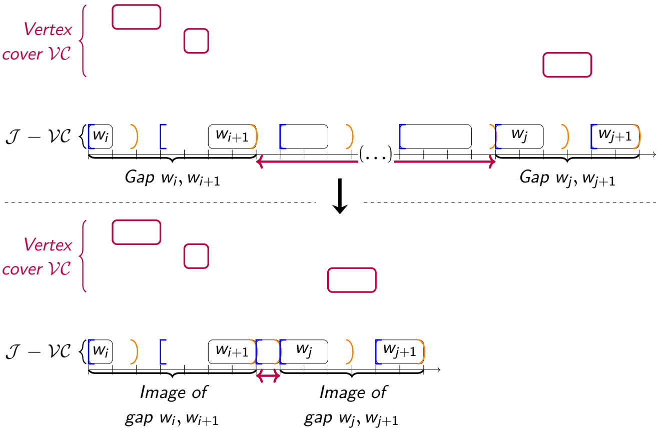

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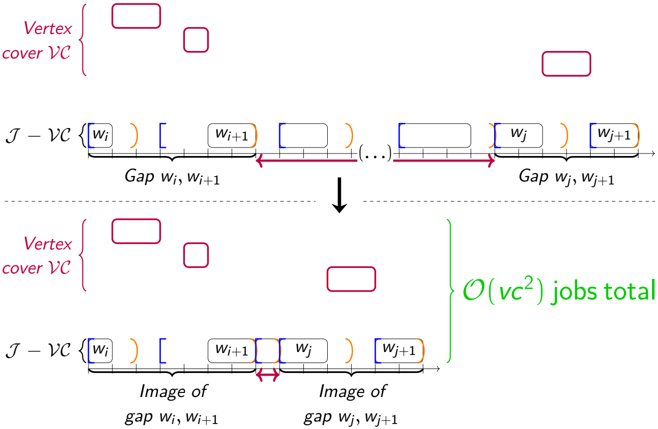

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(2) Reducing time values

Theorem [Frank and Tardos, 1987]

Let $\mathtt{w} = (w_1, \dots, w_d)$ be a vector of rational numbers and let N be a positive integer. Then in time $\mathcal{O}(d^8 \cdot (d + log(N)))$ one can build a vector $\mathtt{w}' = (w_1', \dots, w_d')$ of positive integers such that:

- $w_i' \leq 2^{4d^3} \cdot N^{d(d+2)}$ for all $1 \leq i \leq d$,
- for every $\mathtt{x} = (x_1, \dots, x_d)$ vector of integers such that $\sum_{i=1}^{d} |x_i| \leq N - 1$:

$$\sum_{i=1}^{d} w_i x_i \geq 0 \text{ if and only if } \sum_{i=1}^{d} w_i' x_i \geq 0.$$

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- [Lasserre and Queyranne, 1992] Binary IP equivalent to $1|r_j, d_j|\star$.

$$BIP(I) : \begin{cases} \text{One job per position} & \sum_{j=1}^{n} x_{i,j} = 1 \text{ for all } 1 \leq i \leq n. \\ \text{One position per job} & \sum_{i=1}^{n} x_{i,j} = 1 \text{ for all } 1 \leq j \leq n. \\ \text{Time window checks} & \sum_{j=1}^{n} r_j \cdot x_{i_1,j} + \sum_{i=i_1}^{i_2} \sum_{j=1}^{n} p_j \cdot x_{i,j} \leq \sum_{j=1}^{n} d_j \cdot x_{i_2,j} \\ & \text{for all } 1 \leq i_1 \leq i_2 \leq n. \end{cases}$$

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- $w'_i \leq 2^{4d^3} \cdot N^{d(d+2)}$ for all $1 \leq i \leq d$,
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- Use the theorem on the intermediate instance with $\tilde{n} = \mathcal{O}(vc^2)$ jobs.
  - $d = 3\tilde{n}$, $\mathtt{w} = (p_1, r_1, d_1, \dots, p_{\tilde{n}}, r_{\tilde{n}}, d_{\tilde{n}})$, $N = \tilde{n} + 3$.

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- Kernels on problem $1|r_j, d_j|C_{max}$.
  - Pathwidth pw: no polynomial kernel.
  - Vertex cover number vc: polynomial kernel. ($\mathcal{O}(vc^2)$ jobs, $\mathcal{O}(vc^8)$ size.)

Recap (kernelization)

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Open problems

- Kernel with $\mathcal{O}(vc)$ jobs?
- Polynomial kernel with a smaller parameter than vc?
- Polynomial kernel on $P|prec, p_j = 1, r_j, d_j|C_{max}$?

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- $1|prec, r_j, d_j|C_{max}$: in FPT w.r.t. pathwidth pw.

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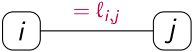
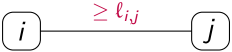
Precedence delays

- $1|prec, r_j, d_j|C_{max}$: in FPT w.r.t. pathwidth pw.
- Given a feasible schedule and $(i \xrightarrow{\ell_{i,j}} j)$:
  - Exact delay:


  - Minimum delay:


  - Maximum delay:


Precedence delays

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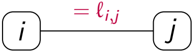
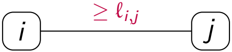


  - Minimum delay:



  - Maximum delay:

- Notation: $prec(\ell^X)$ with $X \in \{ex, min, max\}$.

Precedence delays

- $1|prec, r_j, d_j|C_{max}$: in FPT w.r.t. pathwidth pw.
- Given a feasible schedule and $(i \xrightarrow{\ell_{i,j}} j)$:
  - Exact delay: $\boxed{i} \xrightarrow{= \ell_{i,j}} \boxed{j}$
  - Minimum delay: $\boxed{i} \xrightarrow{\geq \ell_{i,j}} \boxed{j}$
  - Maximum delay: $\boxed{i} \xrightarrow{\leq \ell_{i,j}} \boxed{j}$
- Notation: $prec(\ell^X)$ with $X \in \{ex, min, max\}$.
- [Bodlaender and van der Wegen, 2020] $1|chains(\ell^X), p_j = 1, r_C, d_C|\star$ with $X \in \{ex, min\}$ is $W[t]$-hard parameterized by the thickness for all $t \geq 1$.

$1|chains(\ell^{ex}), p_j = 1, r_j, d_j|C_{max}$ with $pw \leq 2$ is NP-complete.

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- Reduction from 3-COLORING: $G = (V, E)$.

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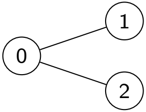
- Reduction from 3-COLORING: $G = (V, E)$.

Example:



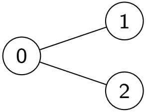
$\longrightarrow$

Scheduling instance with one machine, $3|V| + 2|E|$ jobs and exact delays.

$1|chains(\ell^{ex}), p_j = 1, r_j, d_j|C_{max}$ with $pw \leq 2$ is NP-complete.

- Reduction from 3-COLORING: $G = (V, E)$.

Example:

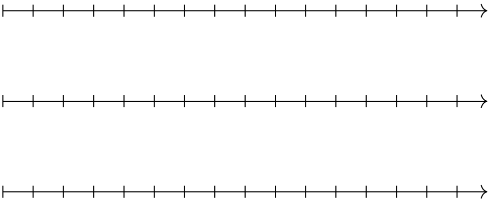


$\longrightarrow$ Scheduling instance with one machine, $3|V| + 2|E|$ jobs and exact delays.

Vertex chain $\mathcal{C}_0$

Vertex chain $\mathcal{C}_1$

Vertex chain $\mathcal{C}_2$



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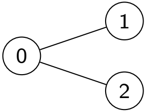

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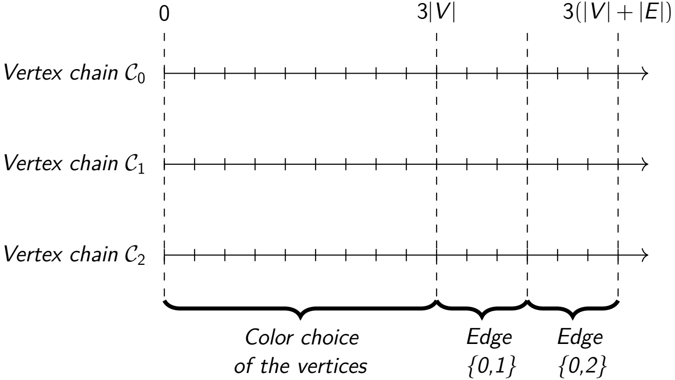

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- Reduction from 3-COLORING: $G = (V, E)$.

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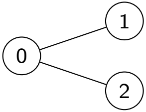

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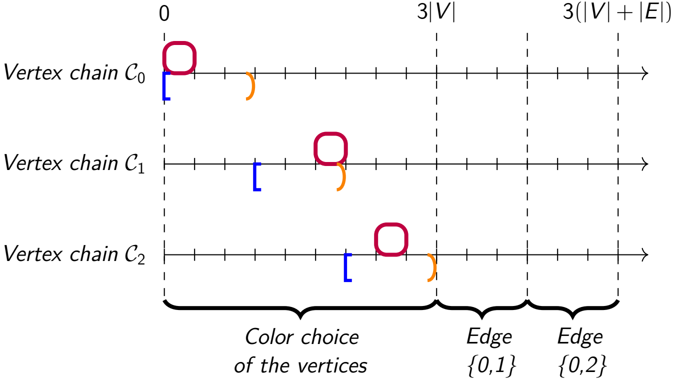

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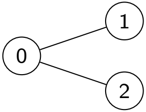
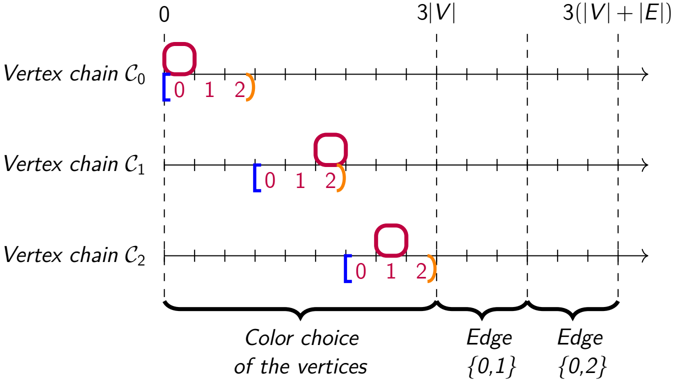


$\longrightarrow$ Scheduling instance with one machine, $3|V| + 2|E|$ jobs and exact delays.



$1|chains(\ell^{ex}), p_j = 1, r_j, d_j|C_{max}$ with $pw \leq 2$ is NP-complete.

- Reduction from 3-COLORING: $G = (V, E)$.

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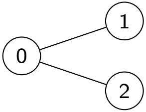

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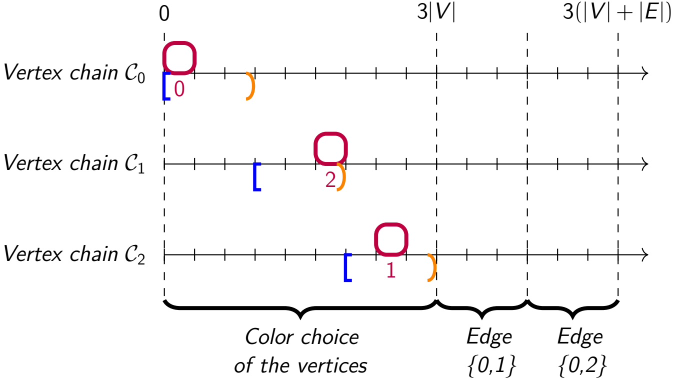

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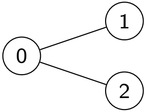

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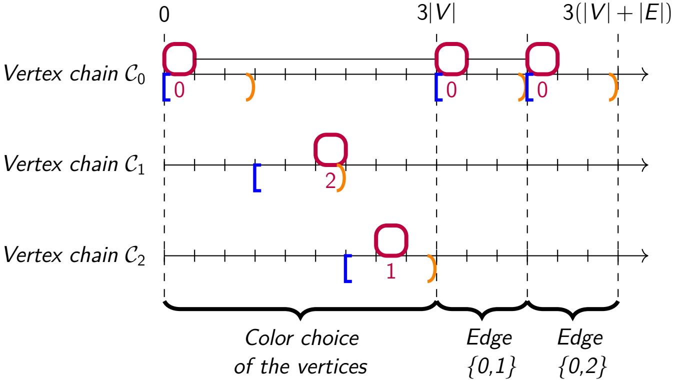

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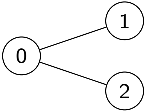

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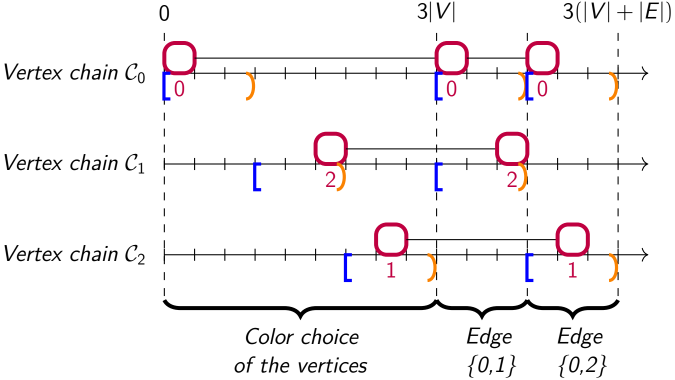

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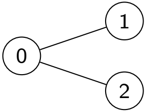

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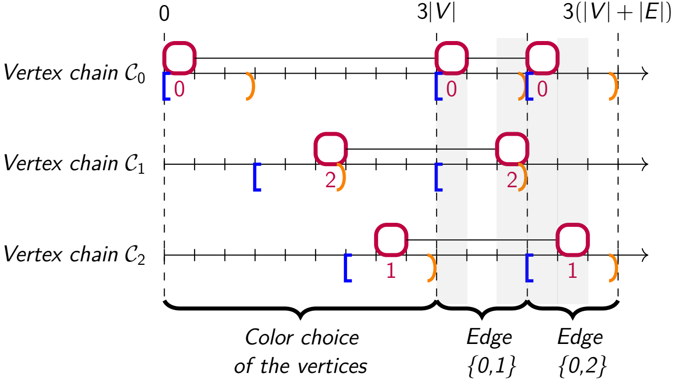

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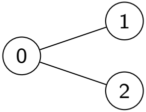

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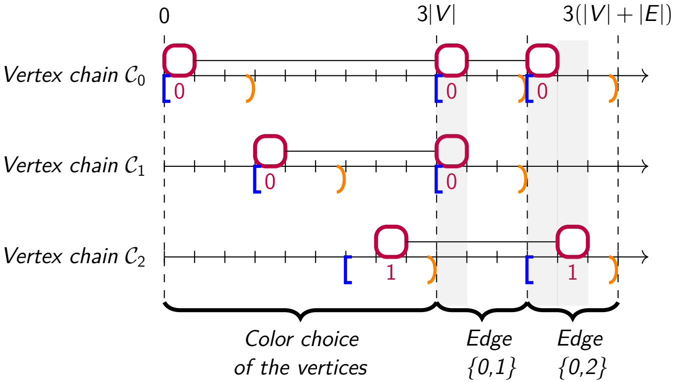

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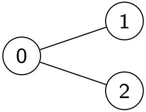

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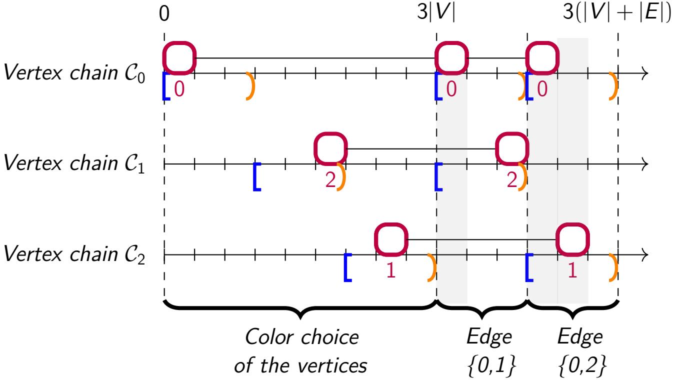

$1|chains(\ell^{min}), p_j = 1, r_j, d_j|C_{max},\ pw \leq 2$ (gadget)

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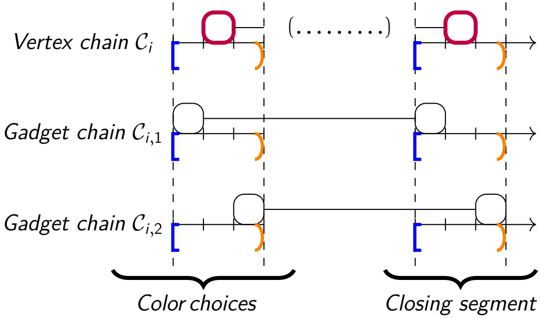

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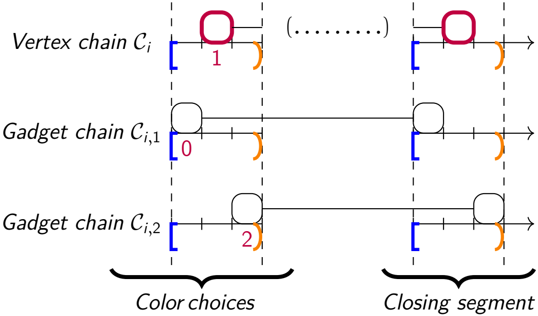

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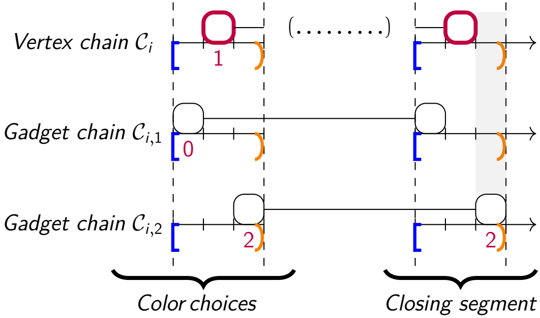

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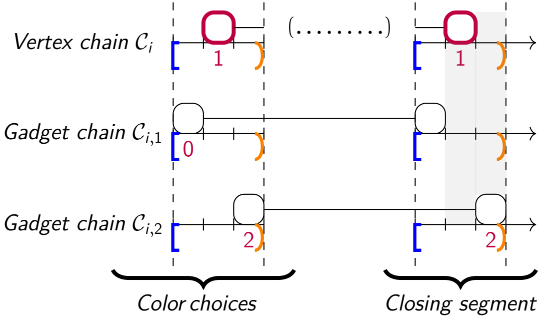

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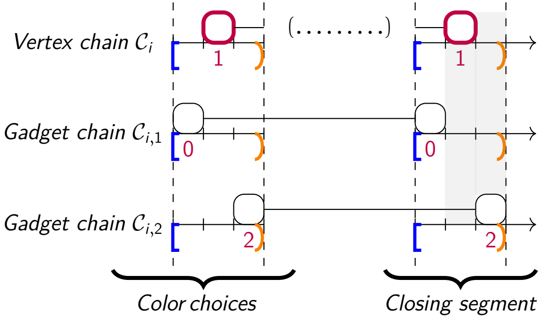

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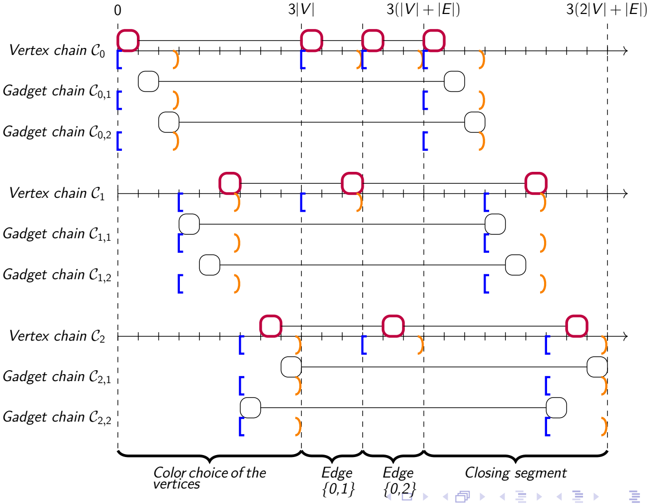
$1|chains(\ell^{min}), p_j = 1, r_j, d_j|C_{max},\ pw \leq 2$ (full reduction)

- Reduction from 3-COLORING: $G = (V, E)$.

Example: $V = \{0, 1, 2\}, E = (\{0, 1\}, \{0, 2\})$.



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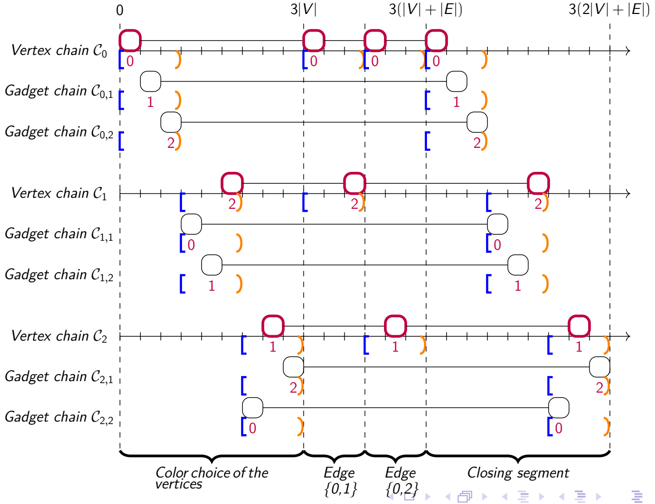

$P|prec(\ell^{min}), p_j = 1, r_j, d_j|C_{max}$ is in FPT w.r.t. $(pw + \ell_{max})$.

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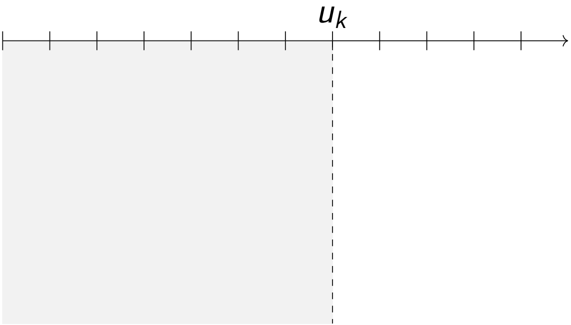

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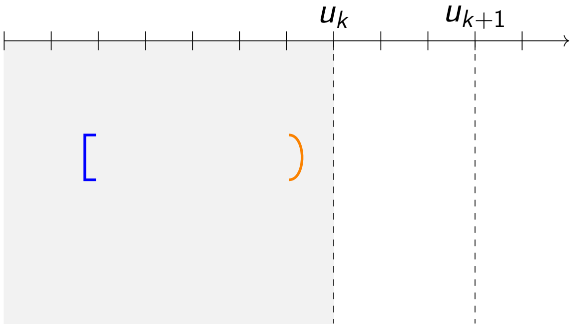

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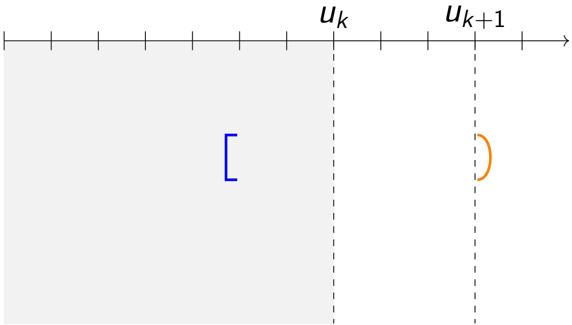

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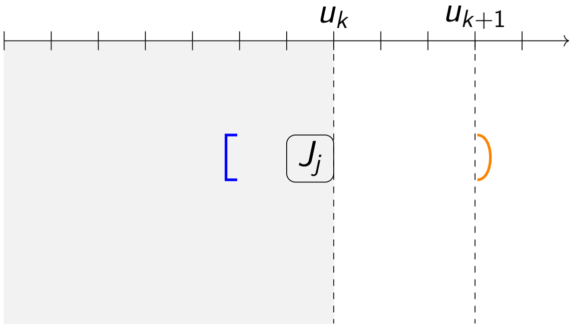

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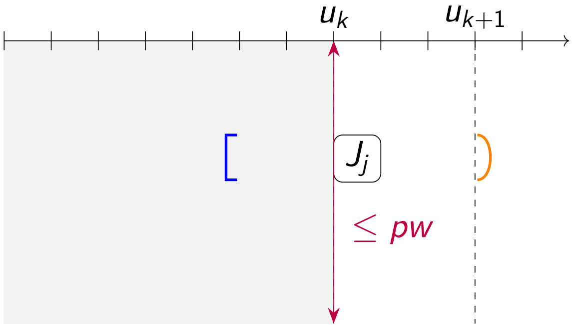

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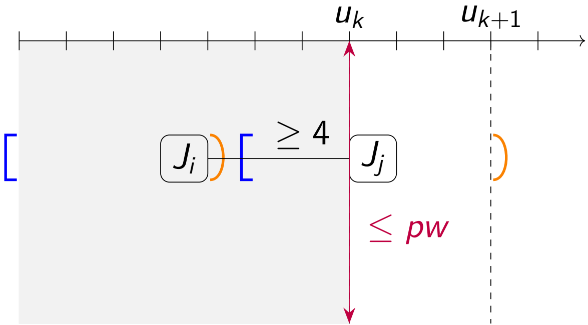

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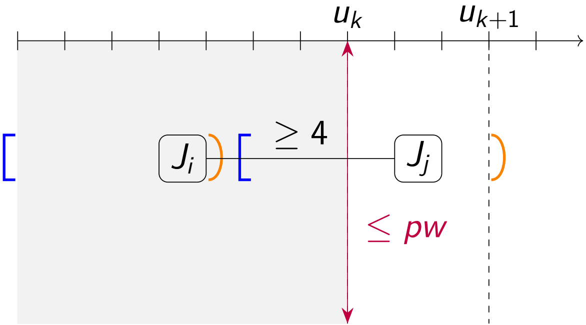

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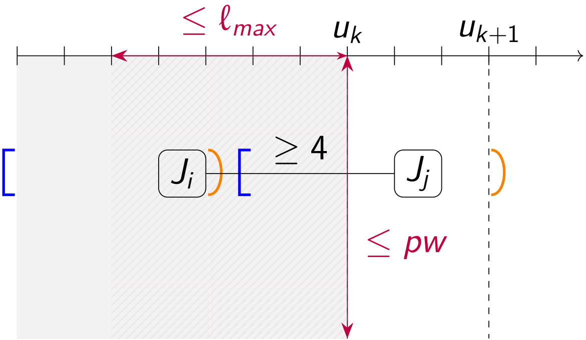

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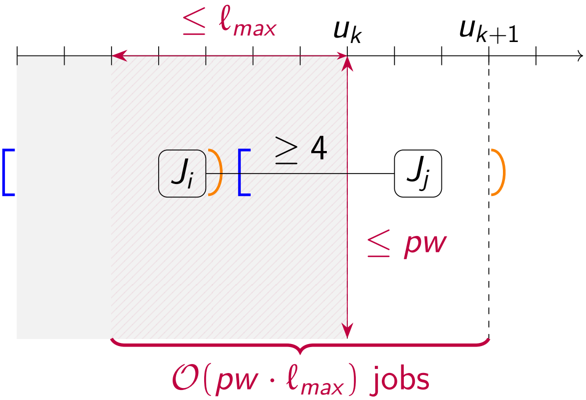

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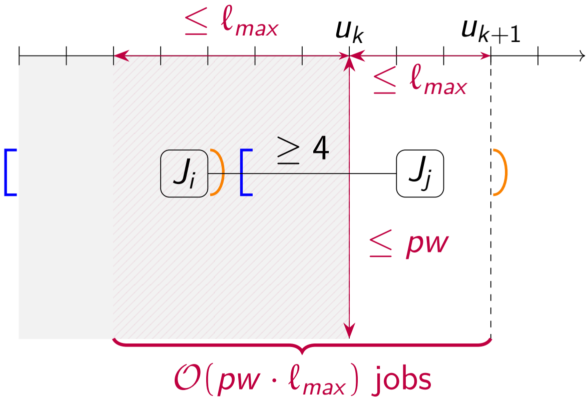

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- $P|prec(\ell^X), p_j = 1, r_j, d_j|C_{max}$ with $X \in \{ex, min\}$.
  - pw: para-NP-complete even with chains on a single machine.
  - $(pw + \ell_{max})$: in FPT (even on general precedence).

Recap (adding precedence delays)

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Open problems

- $1|prec(\ell^{min}), r_j, d_j|C_{max}$.
  - $(pw + \ell_{max})$: in FPT?
  - ℓ_{max}: XNLP-hard. Para-NP-complete?

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- *Topic:* Scheduling radio-astronomical observations.

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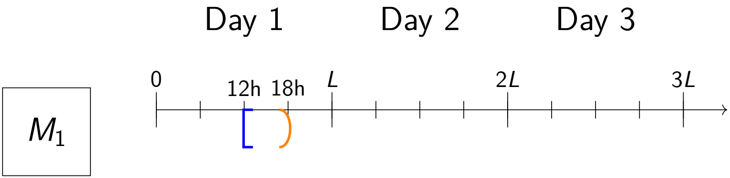

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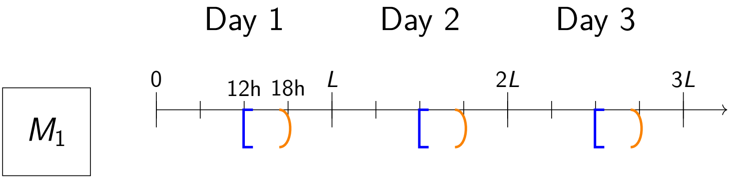

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Algorithm 1: main()

$\triangleright$ Solving $1|r_j, d_j|C_{max}$.

preprocessing() $\triangleright$ Compute sequence $(u_k)_{1 \leq k \leq K}$. Compute associated job subsets.

$u_{K+1} \leftarrow \infty$

Create table T with K columns (1 to K) and 2^{pw+1} slots in each column k.

Initialize T with ∞ everywhere.

$T[1, \emptyset] \leftarrow 0$

for $k = 1$ to $K - 1$ **do**

  for each (L_k, L_{k+1}) **do**

    $T[k+1, L_{k+1}] \leftarrow \min(T[k+1, L_{k+1}], \text{extend}(k, L_k, L_{k+1}))$

return $T[K, \emptyset]$

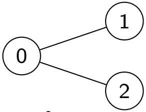
Algorithm 2: extend(k, L_k, L_{k+1})

$Cmax \leftarrow \max(u_k, T[k, L_k])$
if $Cmax = \infty$ **then return** ∞
▷ Ensure that all jobs in L_k with a deadline higher than u_{k+1} are still in L_{k+1}.
if $(L_k - Z_{k+1}) \not\subseteq L_{k+1}$ **then return** ∞
▷ Add jobs to the existing partial schedule.
$mandatory \leftarrow Z_{k+1} \cap \{J_j \in \mathcal{J}, [u_k, u_{k+1}) \subseteq [r_j, d_j)\}$
$add \leftarrow (mandatory \cup L_{k+1}) - L_k$
sort(add, nondecreasing d_j)
for j in add (in order) **do**
    $Cmax \leftarrow Cmax + p_j$
    if $Cmax > d_j$ **then return** ∞
▷ Ensure that all the added jobs start earlier than u_{k+1}.
$last \leftarrow$ last job in add.
if $Cmax - p_{last} \geq u_{k+1}$ **then return** ∞
return $Cmax$

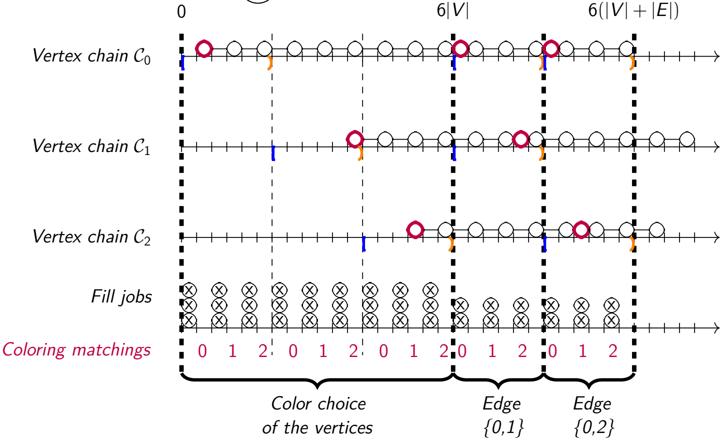
$P|chains(\ell^{ex}), p_j = 1, r_j, d_j|C_{max}$ with $\ell_{max} \leq 1$ is NP-complete.

- Reduction from 3-COLORING: $G = (V, E)$.

Example:



$\longrightarrow$ Scheduling instance with $|V|$ machines and exact delays.



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