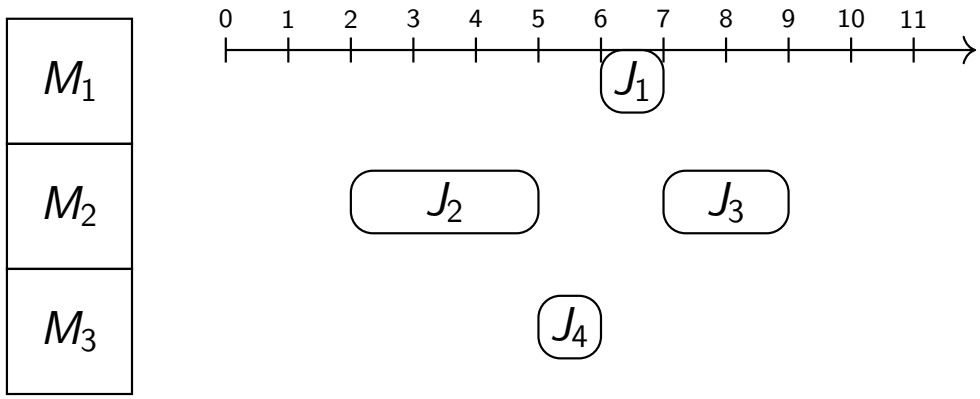
A New Structural Parameter on Single Machine Scheduling with Release Dates and Deadlines

Maher Mallem, Claire Hanen, Alix Munier-Kordon

24 May 2024

Machine scheduling



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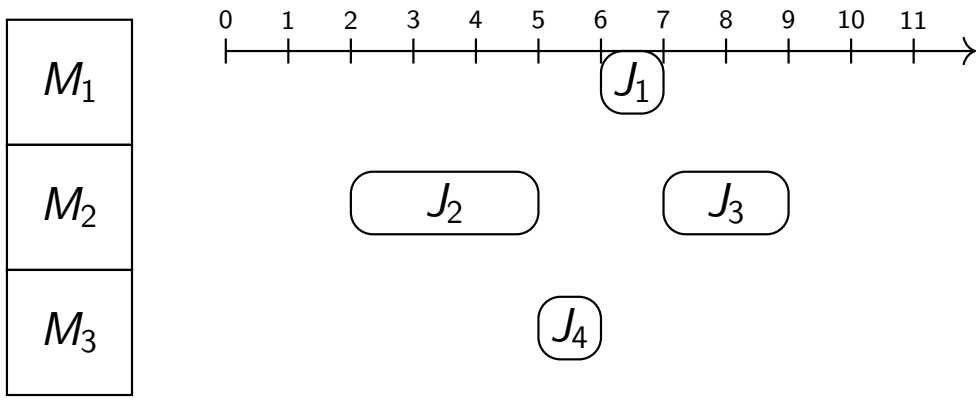
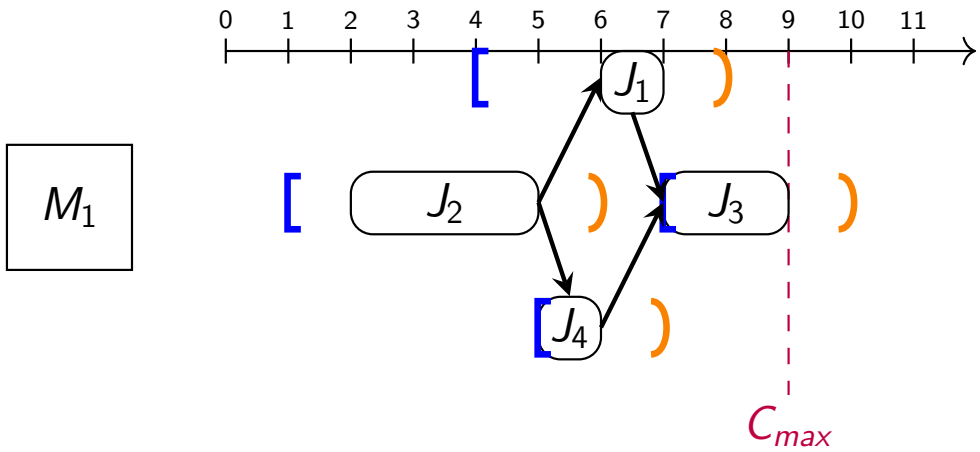


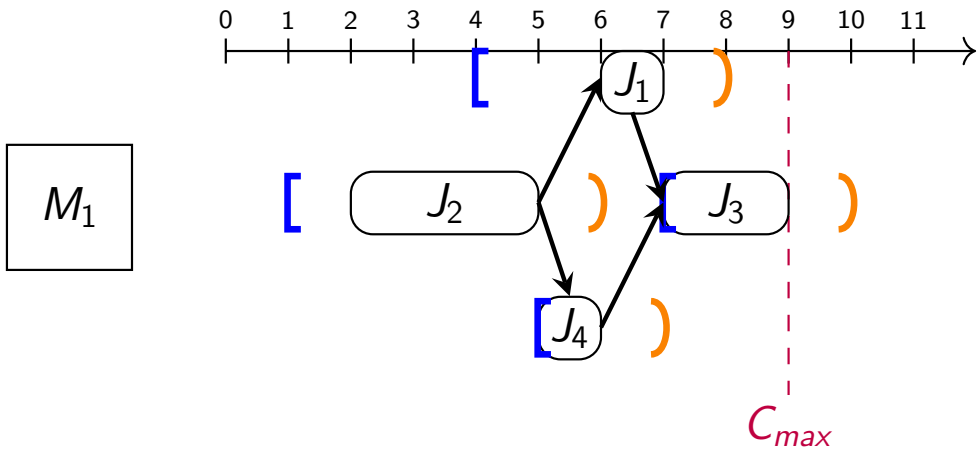
machines | job properties | optimization criterion

Machine scheduling



$1|prec, r_j, d_j|C_{max}$

Machine scheduling



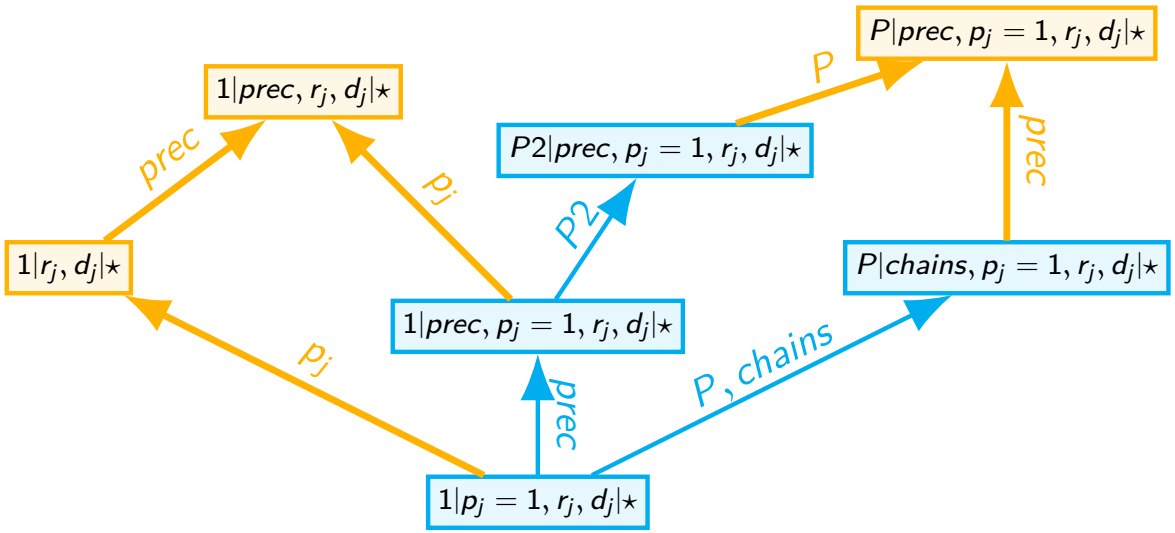
$1|prec, r_j, d_j|C_{max}$

$1|r_j, d_j|\star$ is strongly NP-hard. [Brucker, Kan, Lenstra 77]

Classical complexity theory

$1|p_j = 1, r_j, d_j|\star$

Classical complexity theory



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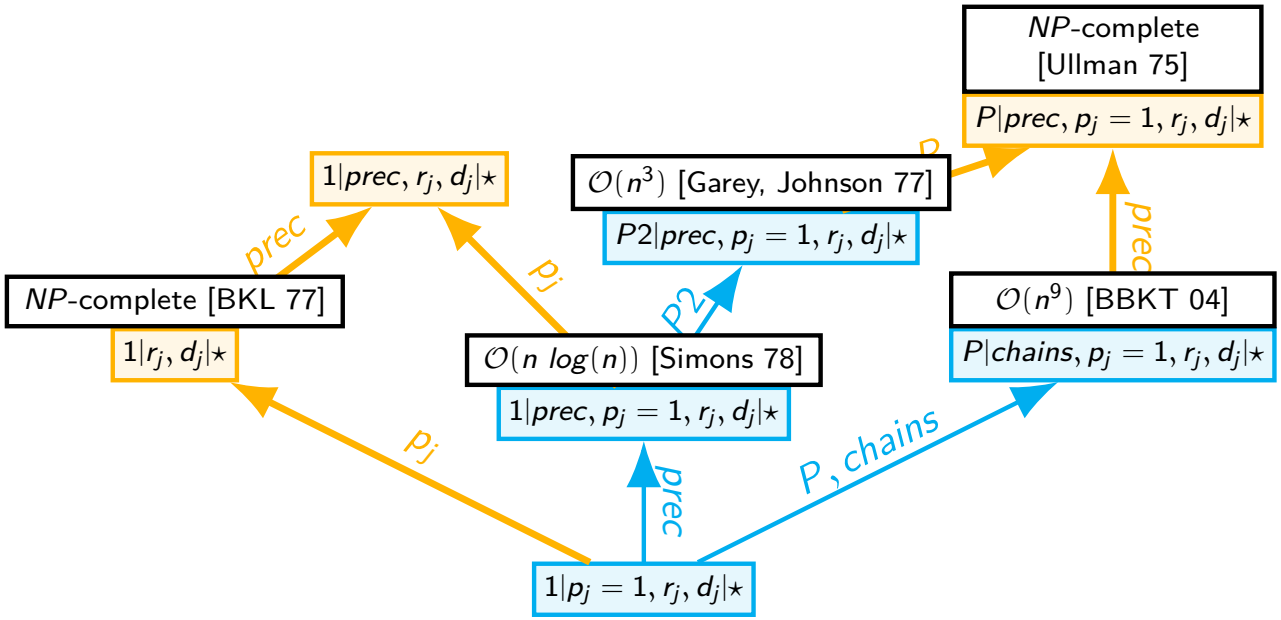

Going beyond NP-hardness

- Problem $\mathcal{P}$
- P: deterministic time $poly(n)$
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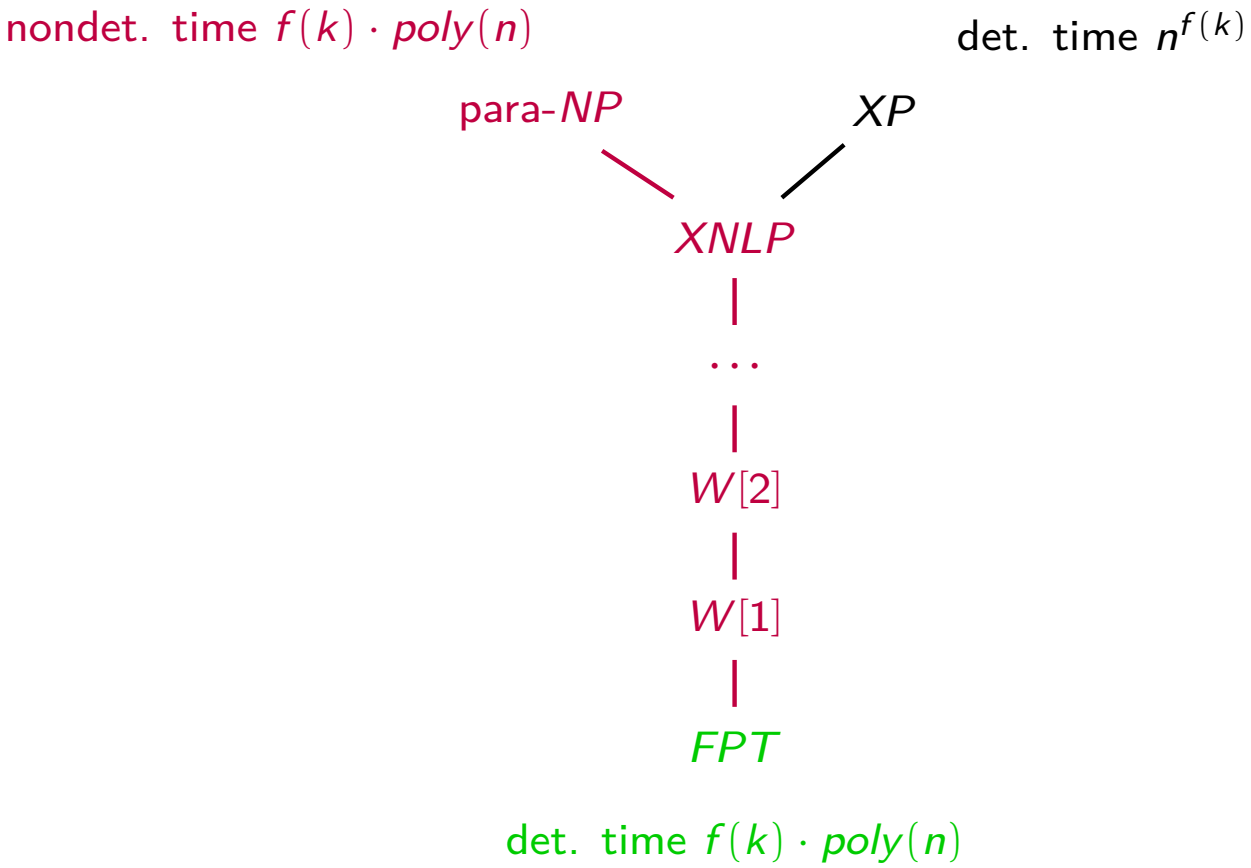
Going beyond NP-hardness

- Problem $\mathcal{P}$
  - Problem $(\mathcal{P}, k)$
- P: deterministic time $poly(n)$
  - FPT: deterministic time $f(k) \cdot poly(n)$
- NP: nondeterministic time $poly(n)$
  - para-NP: nondeterministic time $f(k) \cdot poly(n)$

Parameterized complexity classes



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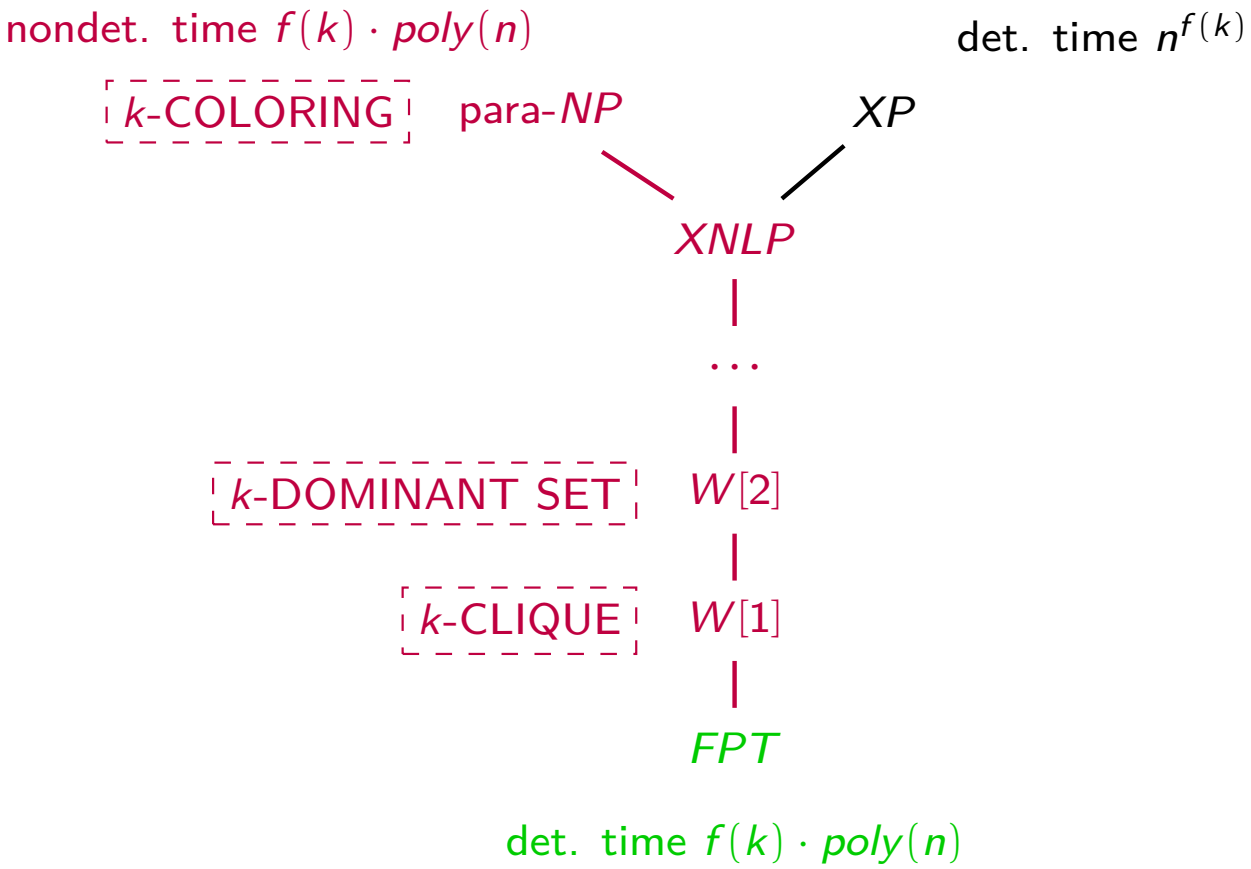

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  - [Lenstra 83] MILP
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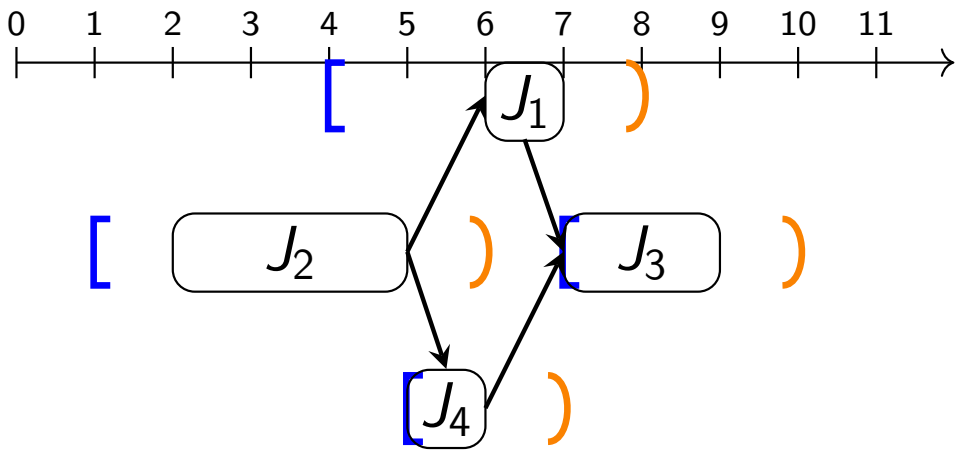
- Dynamic Programming

Pathwidth μ



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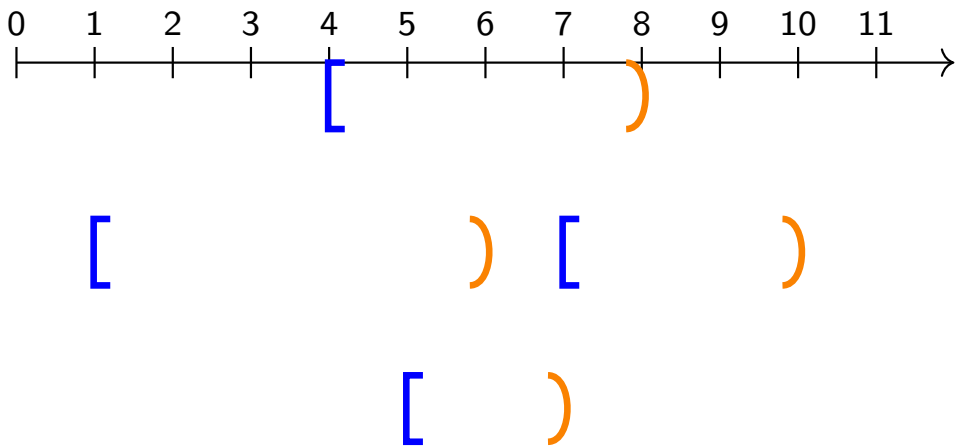

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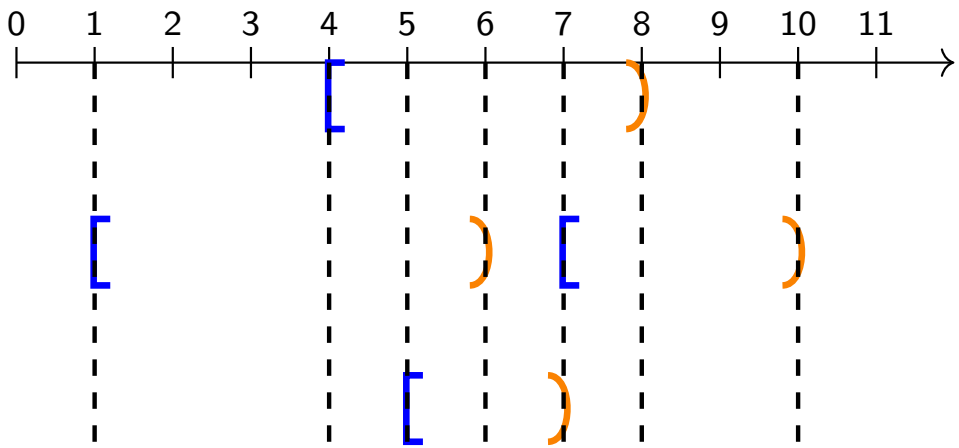

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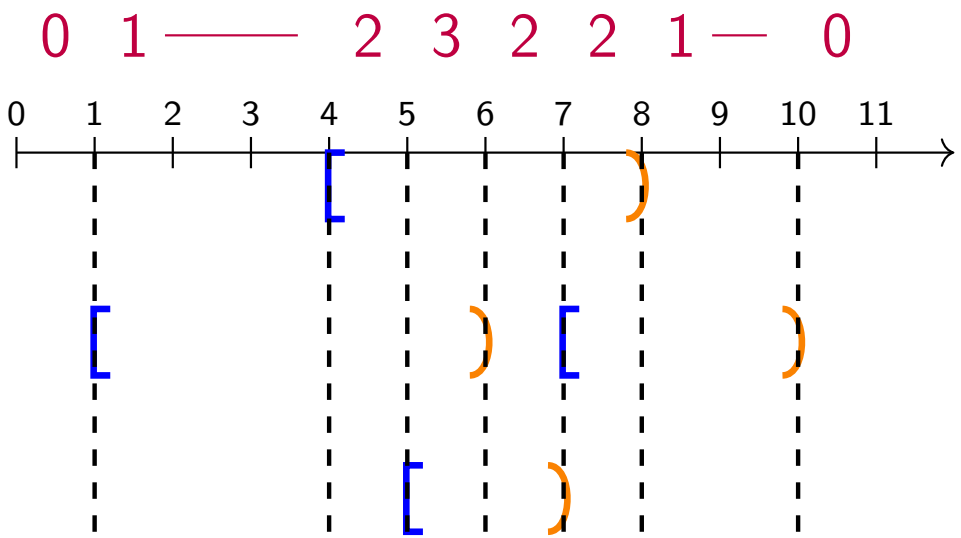

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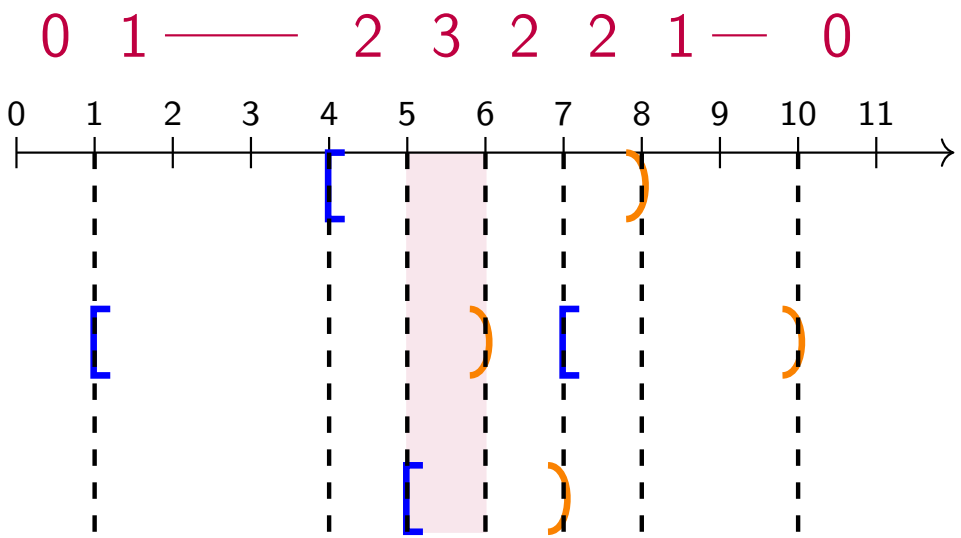
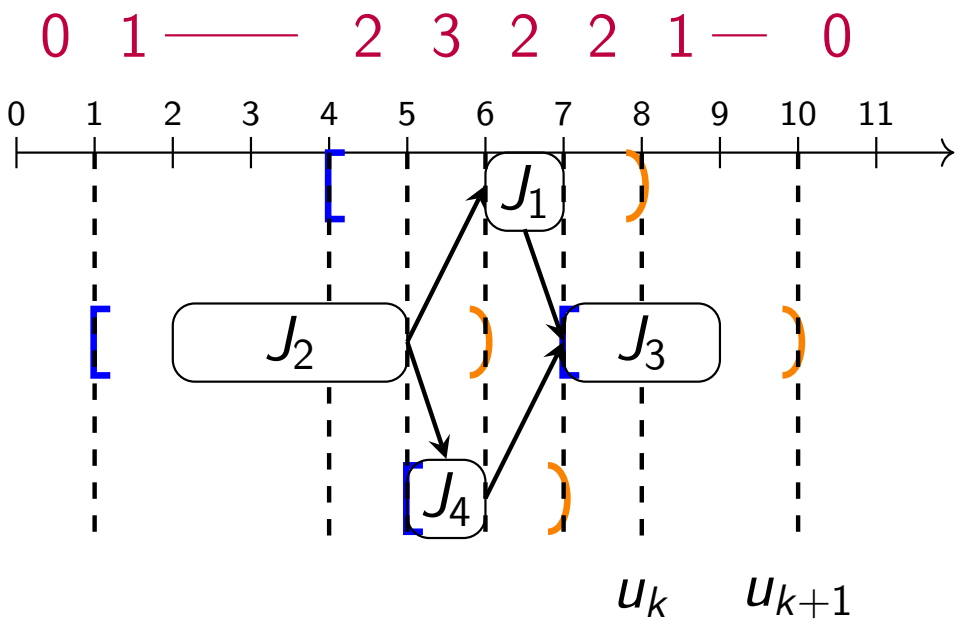


$$\mu = 3 - 1 = 2$$

Pathwidth μ



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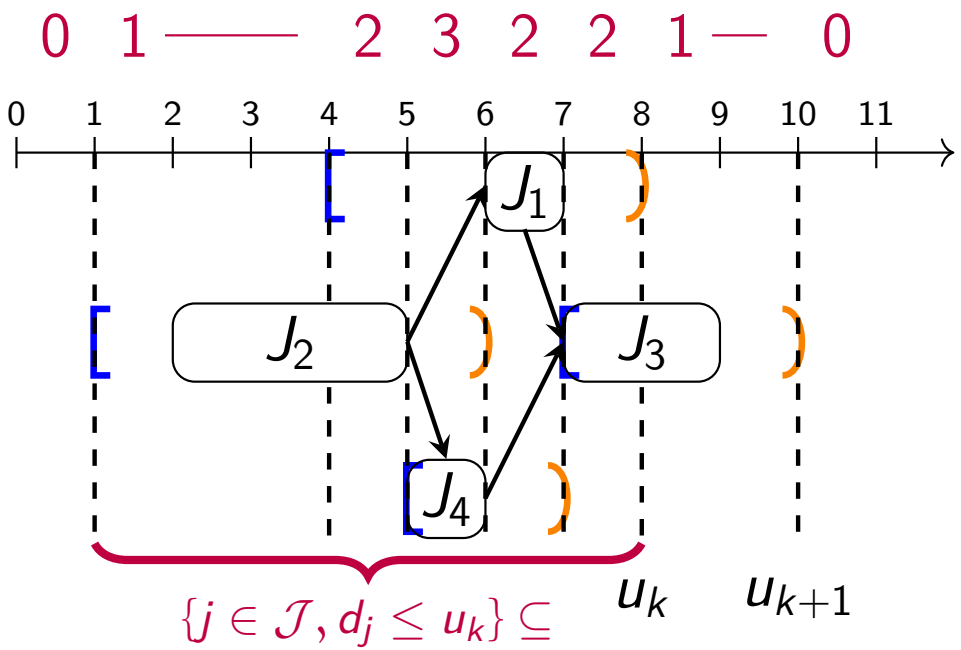

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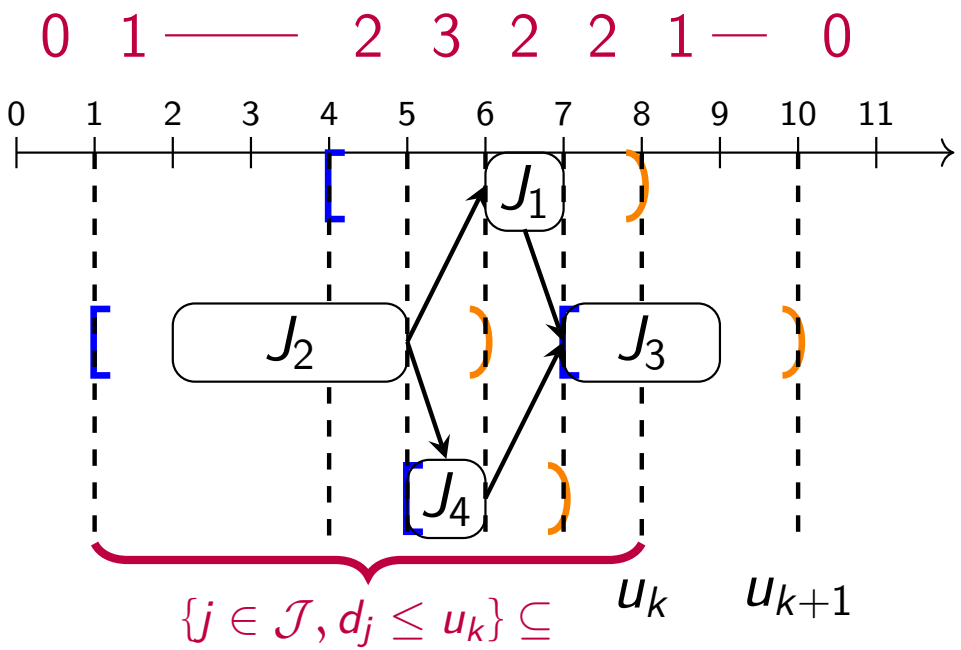

$\leq 2^{\mu+1}$ states per time segment $[u_k, u_{k+1})$.

Results with parameter μ

- [Munier 21]
  - $P|prec, p_j = 1, r_j, d_j|C_{max}$
  - Exact resolution in time $\mathcal{O}(16^{\mu} \cdot n^4)$.
- [Baart, He, de Weerdt 21]
  - $1|r_j, s_{jk}, reject, \bar{d}_j| \sum_{j \notin R}(v_j - w_j T_j)$
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- [M., Hanen, Munier 22]
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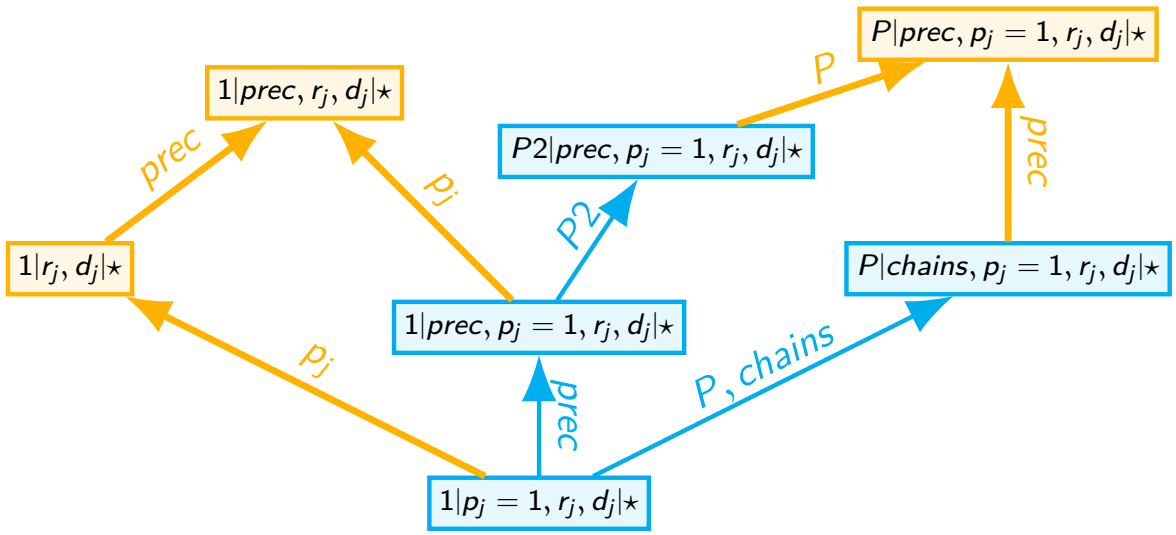
Parameter μ



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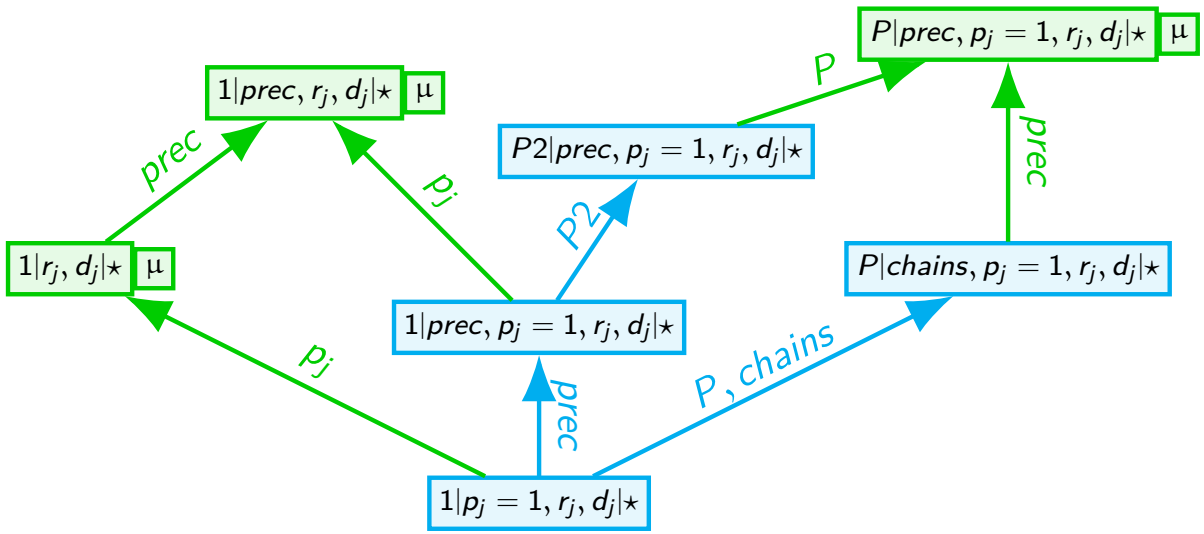

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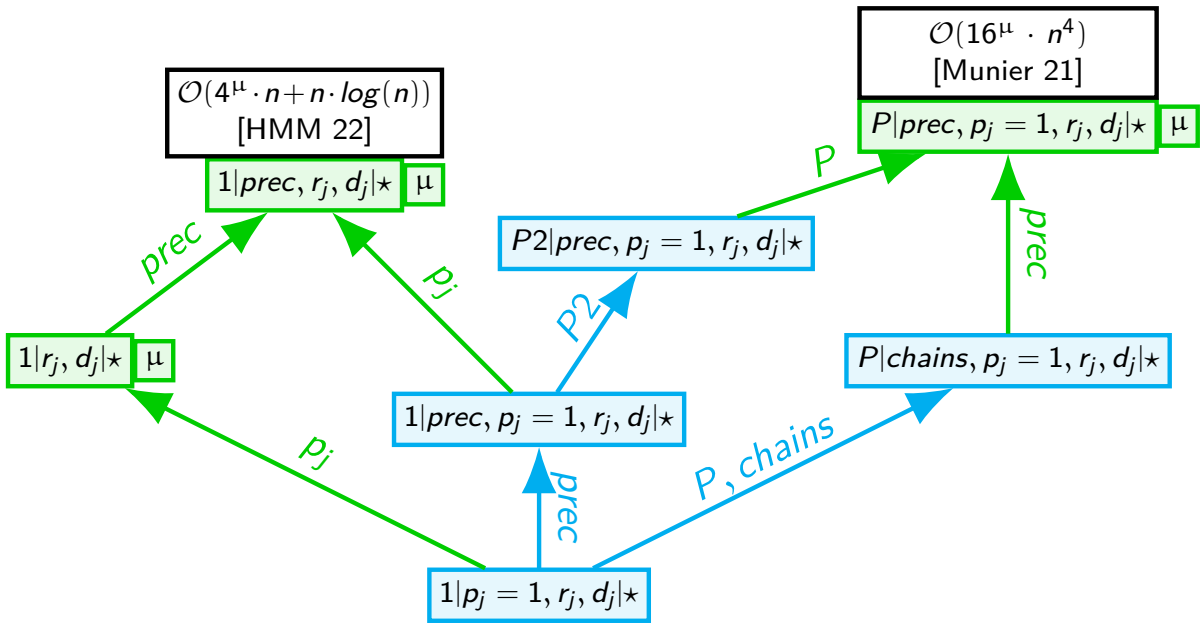

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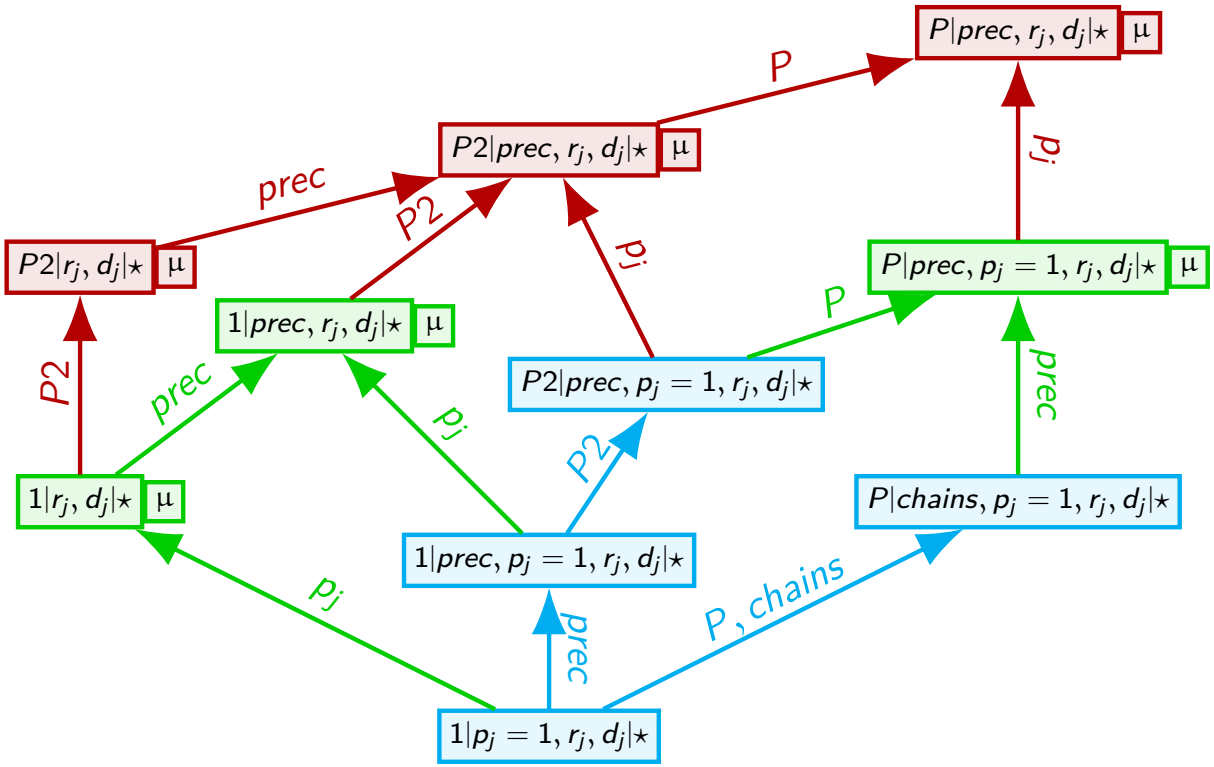

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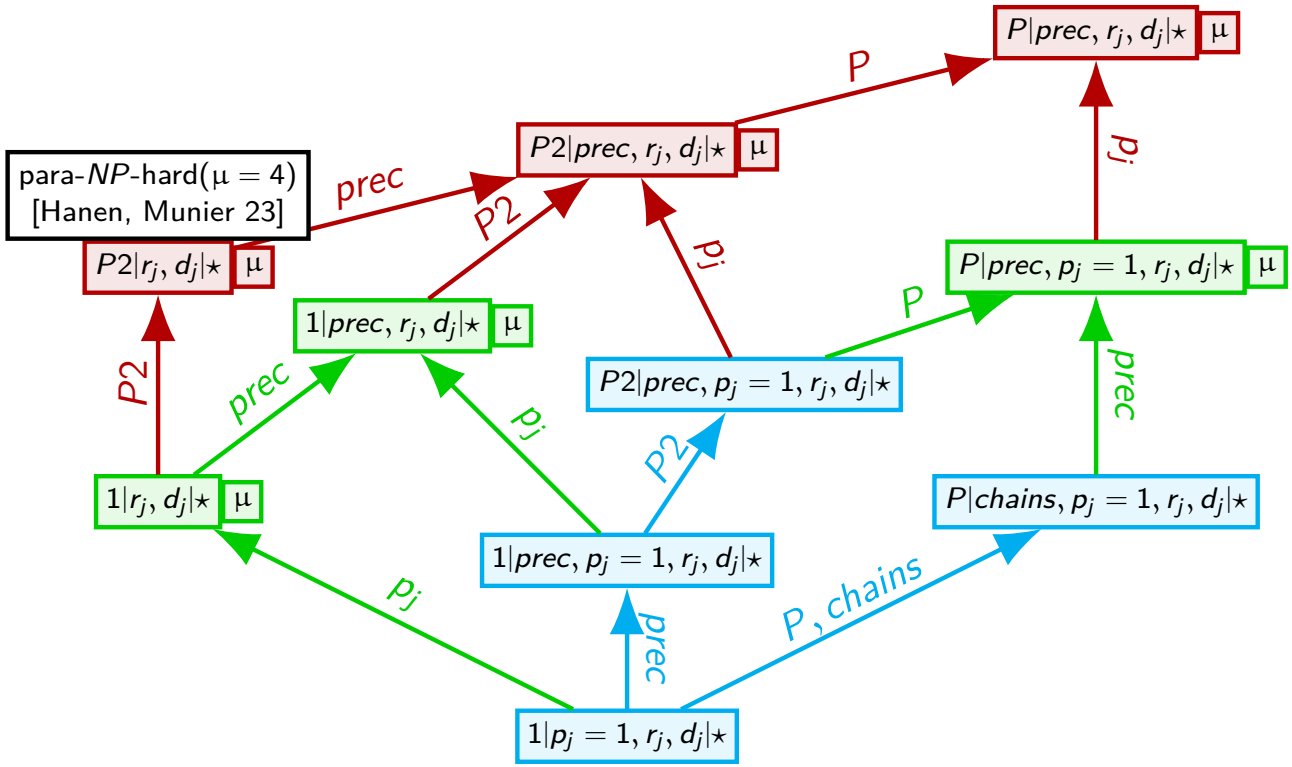
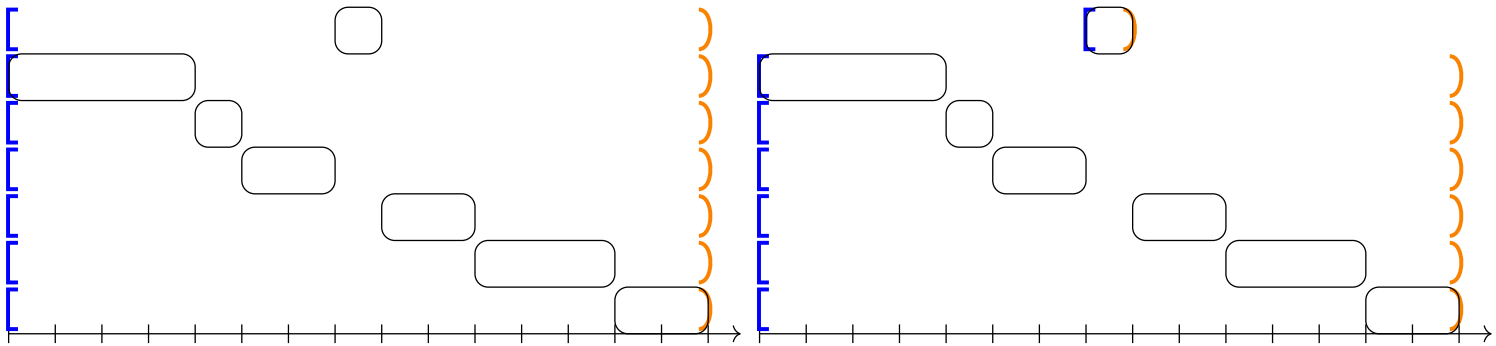


Pathwidth limitation

- Two instances of $1|r_j, d_j|\star$.



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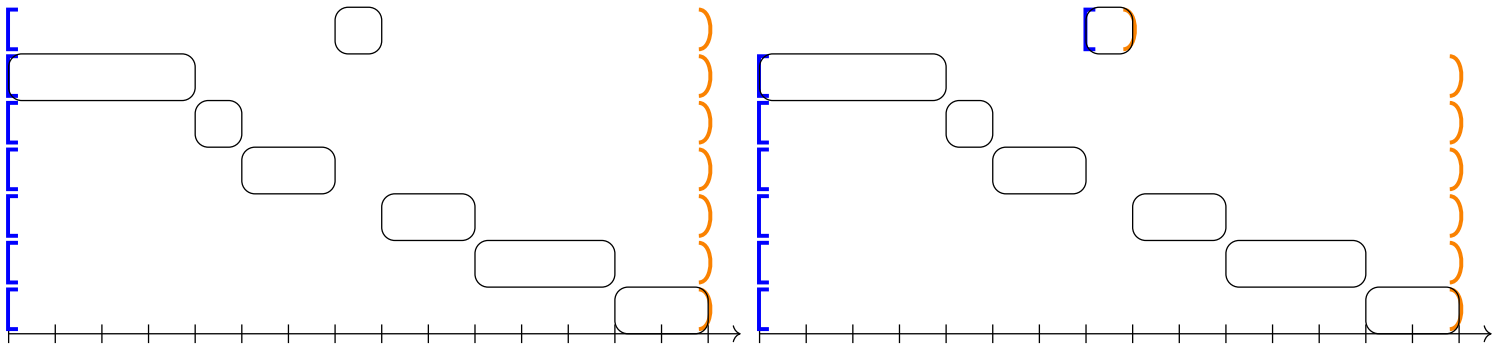

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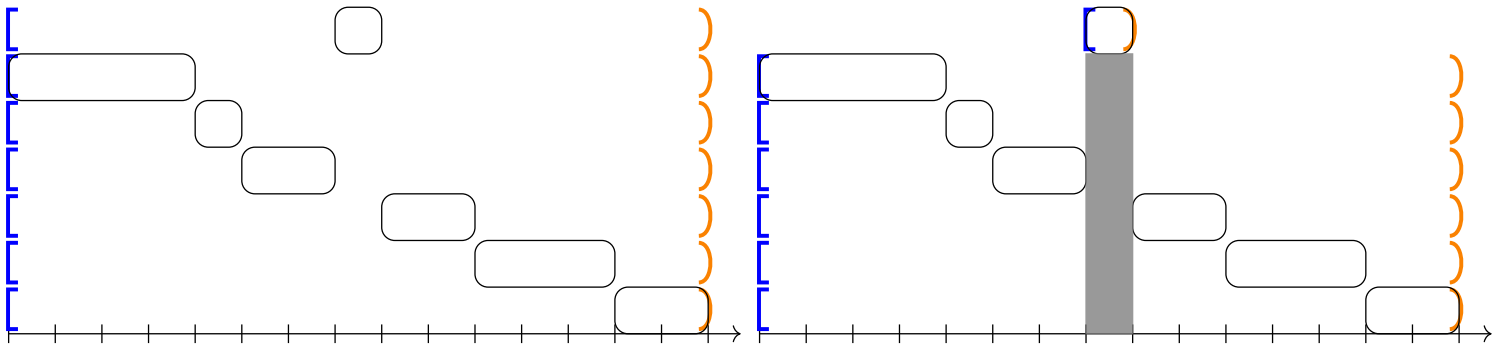

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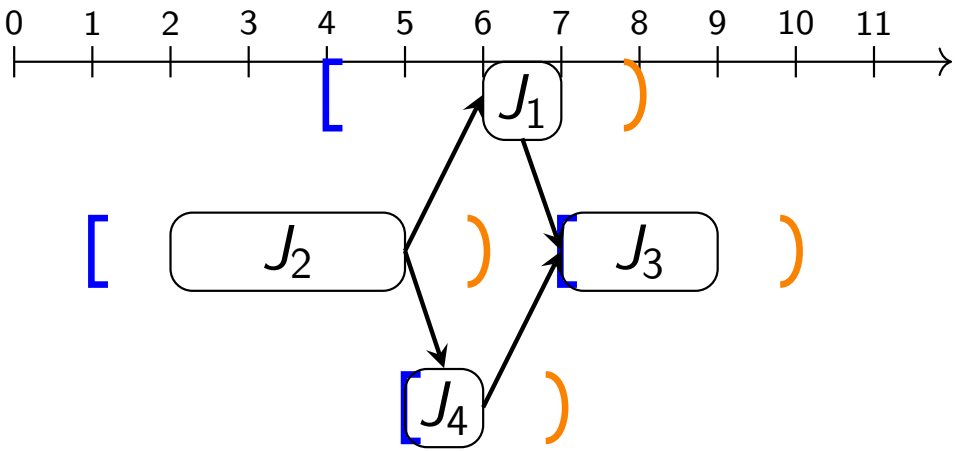
Proper level q



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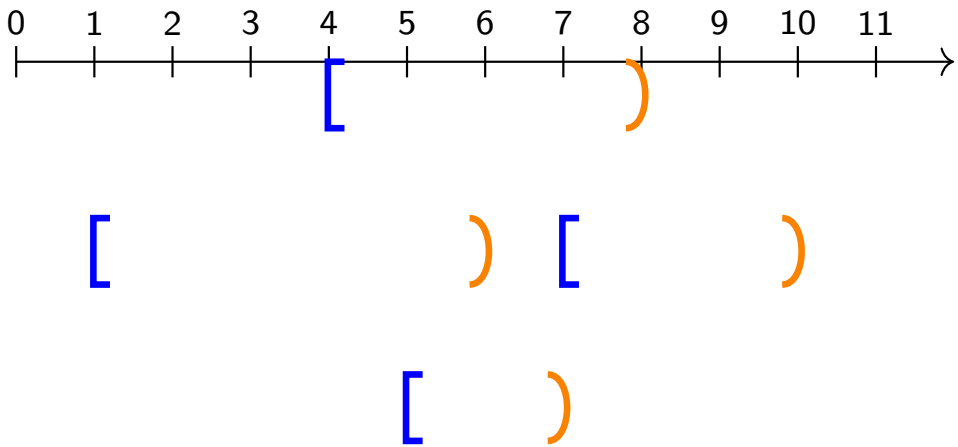

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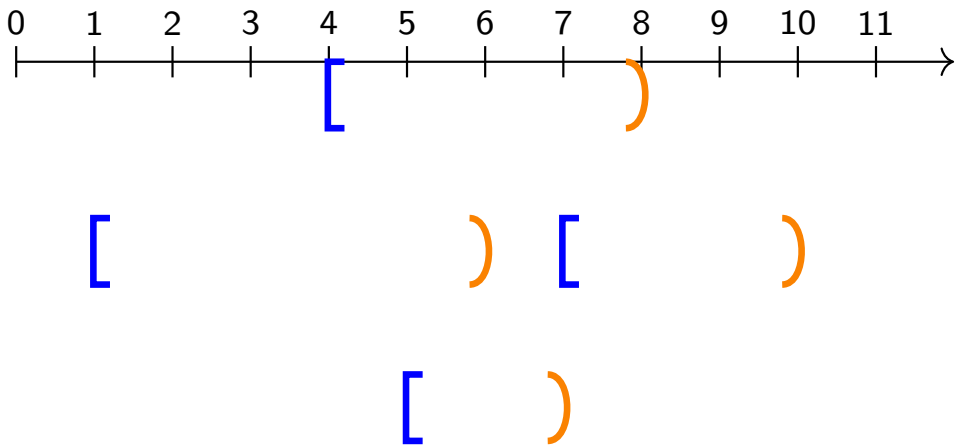

Definition: Proper sets and proper level

- $\forall i \in \mathcal{J}, \Pi_i = \{j \in \mathcal{J}, r_j < r_i \wedge d_i < d_j\}$
- $q = max_{i \in \mathcal{J}} |\Pi_i|$

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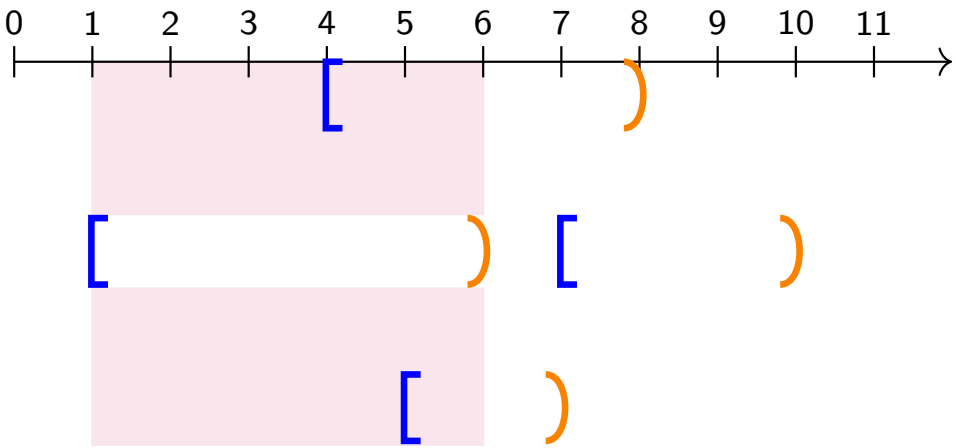

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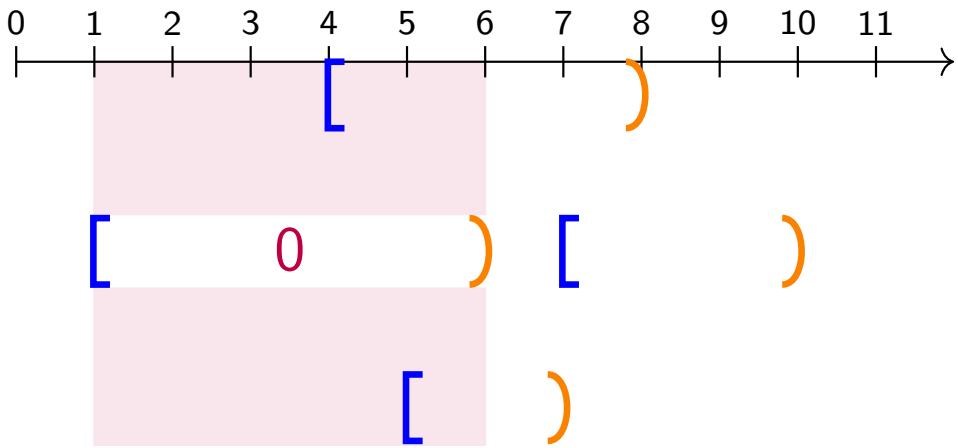

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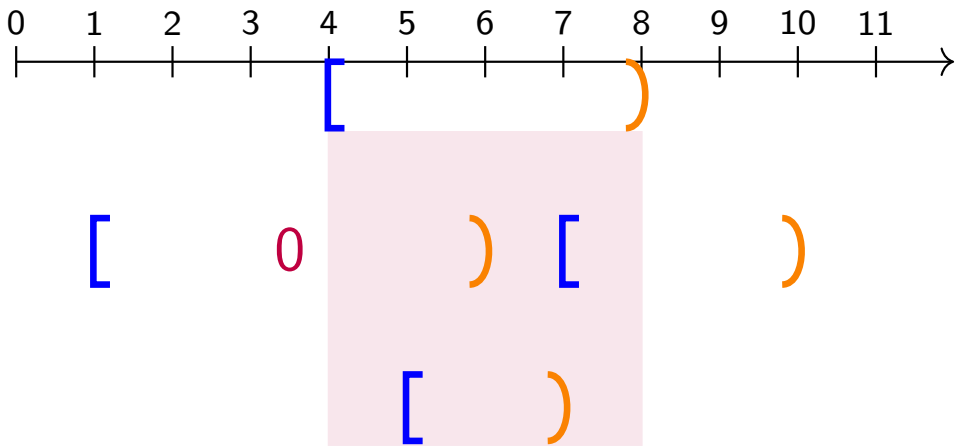

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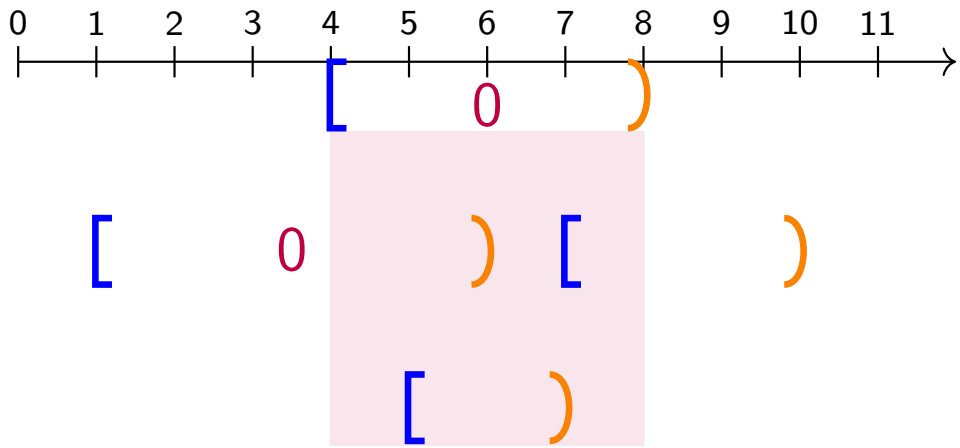

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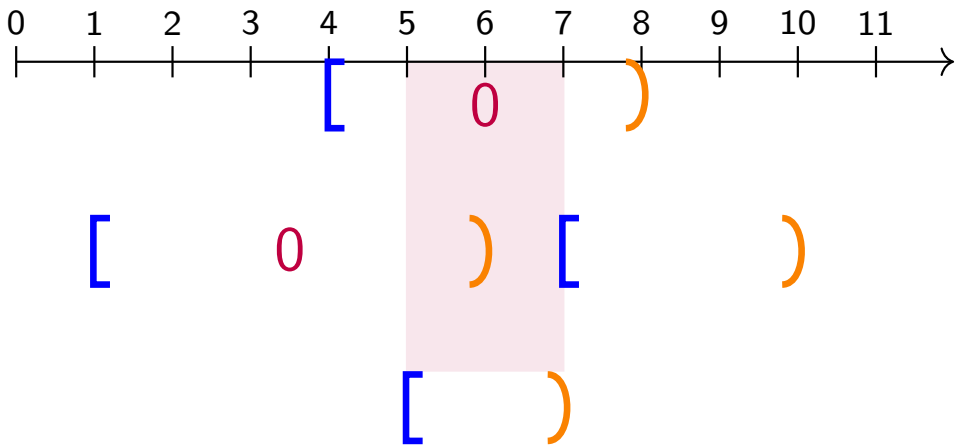

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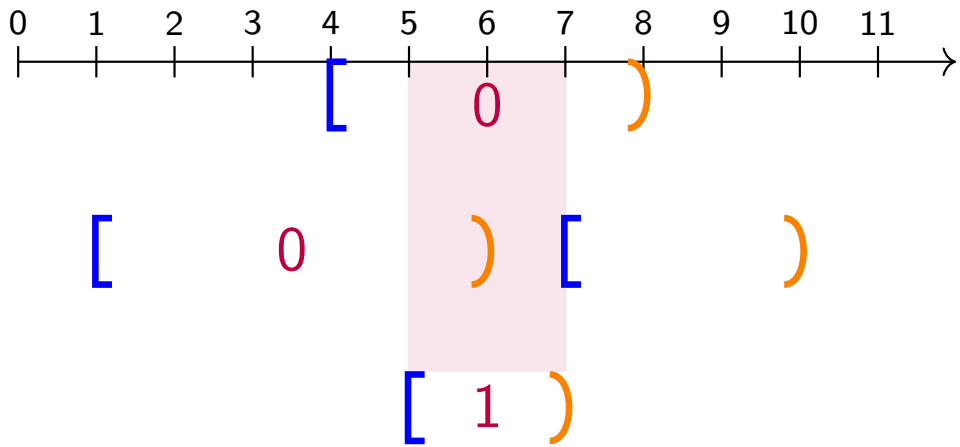

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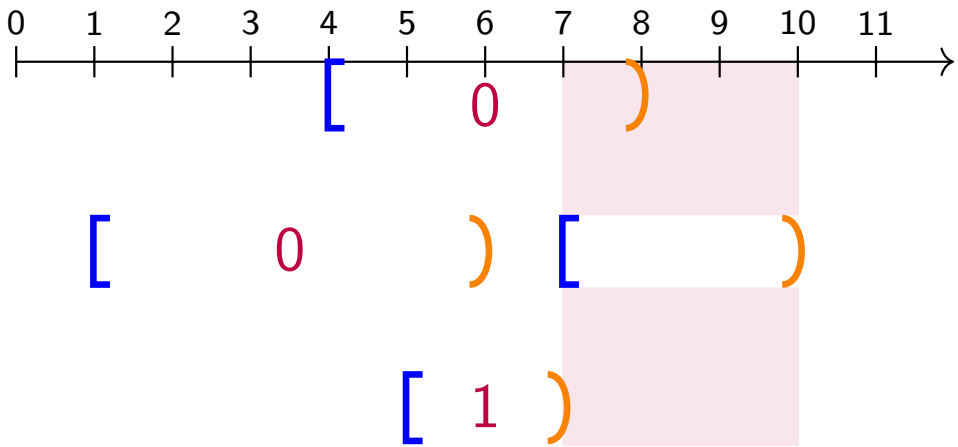

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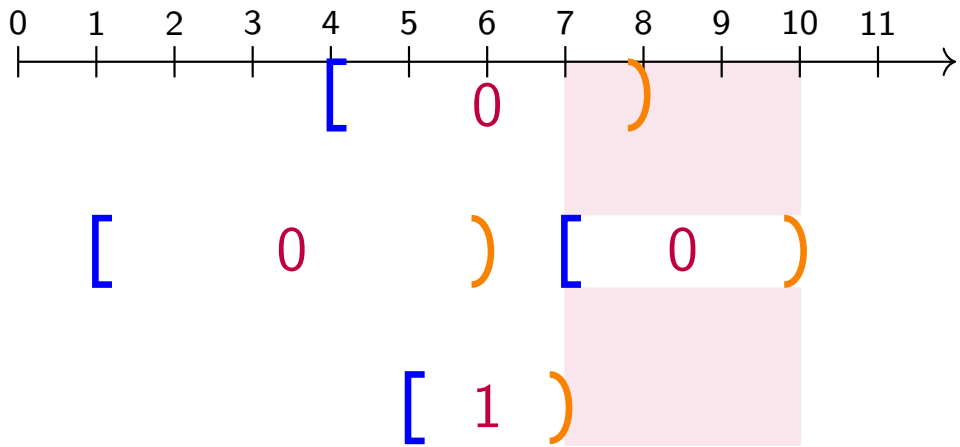

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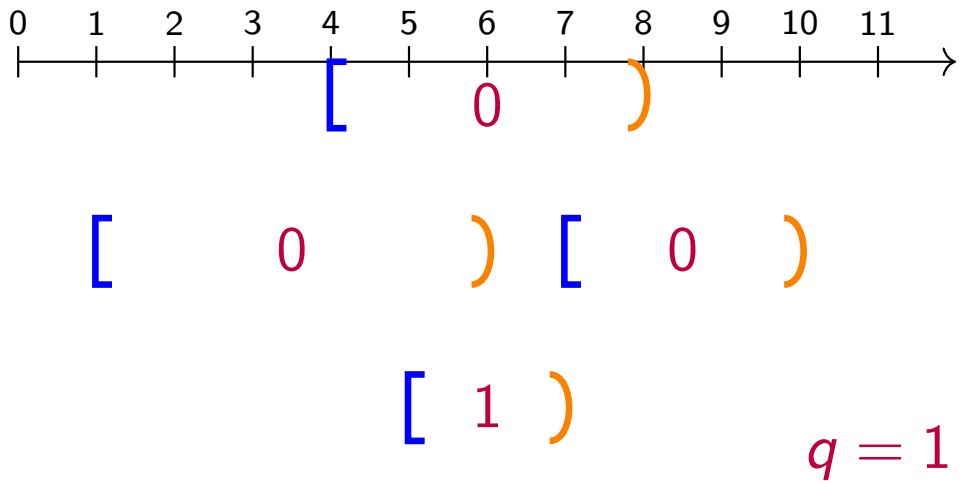

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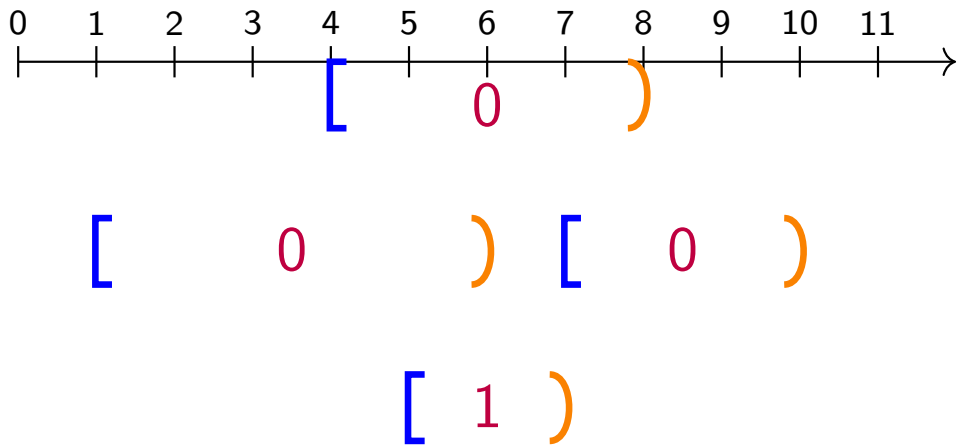

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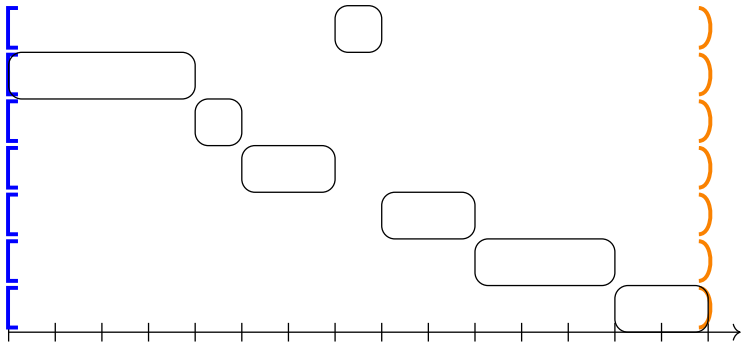
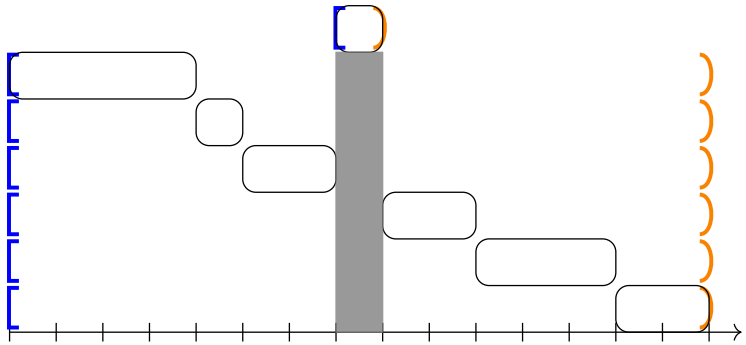
- $\forall i \in \mathcal{J}, \Pi_i = \{j \in \mathcal{J}, r_j < r_i \wedge d_i < d_j\}$ [Erschler, Fontan, Merce 85]
- $q = max_{i \in \mathcal{J}} |\Pi_i|$ [Proskurowski, Telle 99]

Pathwidth vs proper level



$\mu = n - 1$



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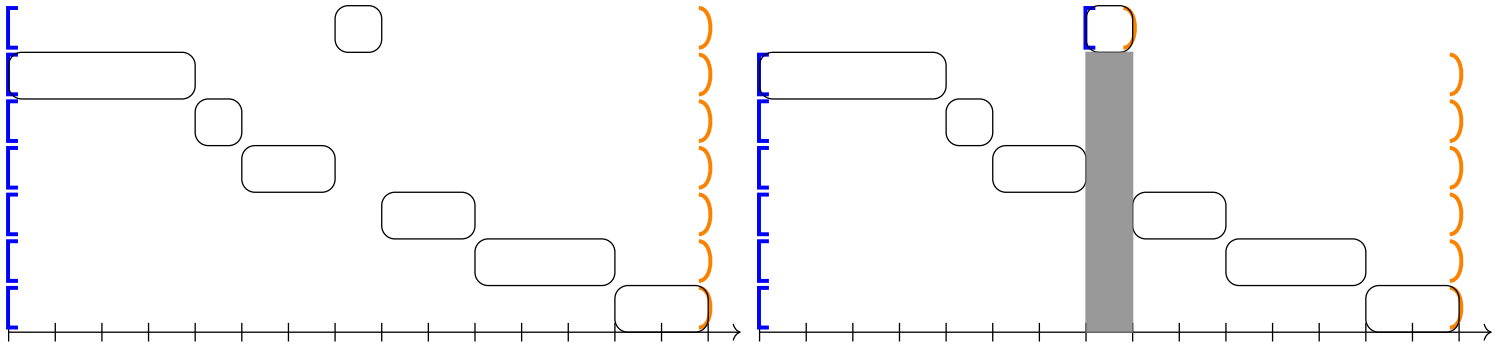

$\mu = n - 1$

$q = 0$

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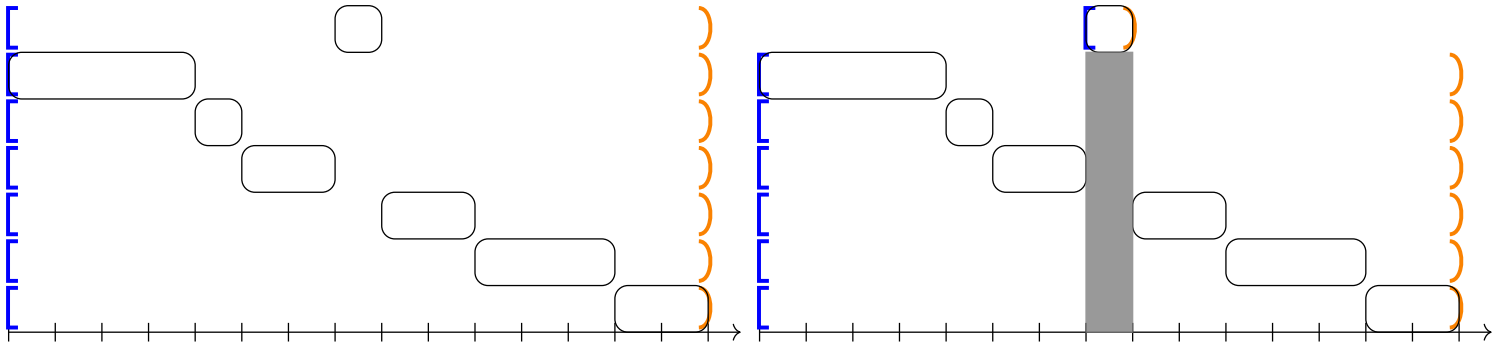
Pathwidth vs proper level



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Result

$$q \leq \mu$$

Extending the Earliest Deadline rule

- σ optimal schedule on $1|r_j, d_j|C_{max}$ ($=$ job ordering).
- $\prec$ $=$ lexicographic order (non decreasing d_j, non decreasing r_j) on $\mathcal{J}$.

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Definition: Earliest Deadline rule (ED) [Lawler, Moore 69]

Schedule σ follows the (ED) rule if:

$$\forall (i,j) \in \mathcal{J}^2, i \prec j \implies \sigma(i) < \sigma(j).$$

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Definition: weak Earliest Deadline rule (wED)

Schedule σ follows the (wED) rule if:

$$\forall (i,j) \in \mathcal{J}^2, i \prec j \implies [\sigma(i) < \sigma(j)] \vee [j \in \Pi_i] \vee [\exists h \prec i, \sigma(j) < \sigma(h) < \sigma(i)].$$

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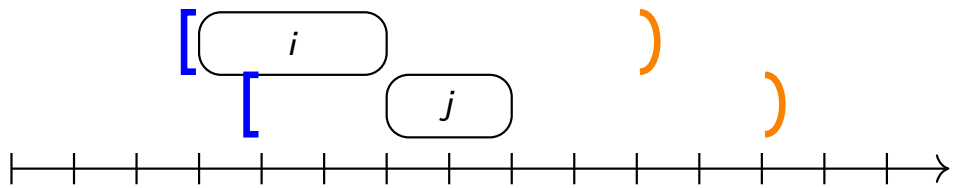

Extending the Earliest Deadline rule

- σ optimal schedule on $1|r_j, d_j|C_{max}$ (= job ordering).
- $\prec$ = lexicographic order (non decreasing d_j, non decreasing r_j) on $\mathcal{J}$.

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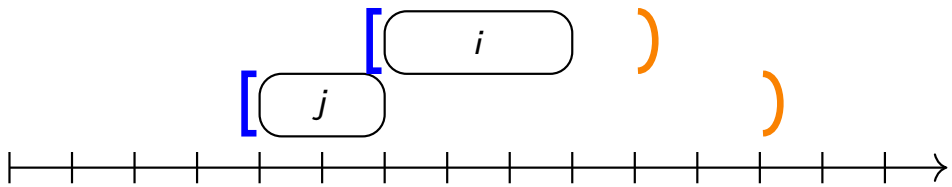

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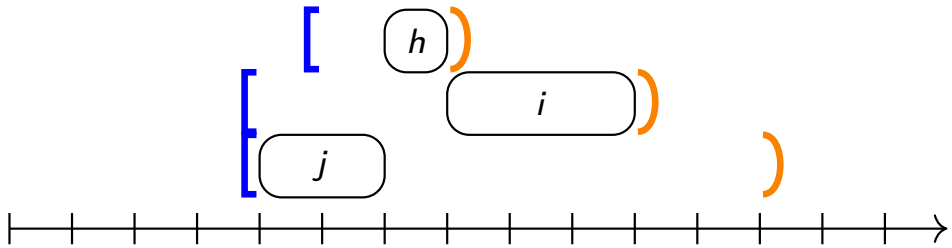

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Proposition

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A summit is a job i in $\mathcal{J}$ such that: $\forall j \in \mathcal{J}, i \notin \Pi_j$.

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- Rename jobs in $[1, n]$ following their lexicographic order $\prec$.

- Schedule σ up to summit s_k: set $[1, s_k] \cup L_k$ with $L_k \subseteq \Pi_{s_k}$.
- σ of the form $T_1 s_1 \ldots s_k T_{k+1} s_{k+1} \ldots s_N T_{N+1}$.

Summit-based dominance

Lemma 2: Summit-based decomposition

Any feasible schedule σ which follows the (wED) rule is of the form $T_1 s_1 \dots s_k T_{k+1} s_{k+1} \dots s_N T_{N+1}$ where every job in sequence T_{k+1} is in set $\Pi_{s_k} \cup \Pi_{s_{k+1}}$.

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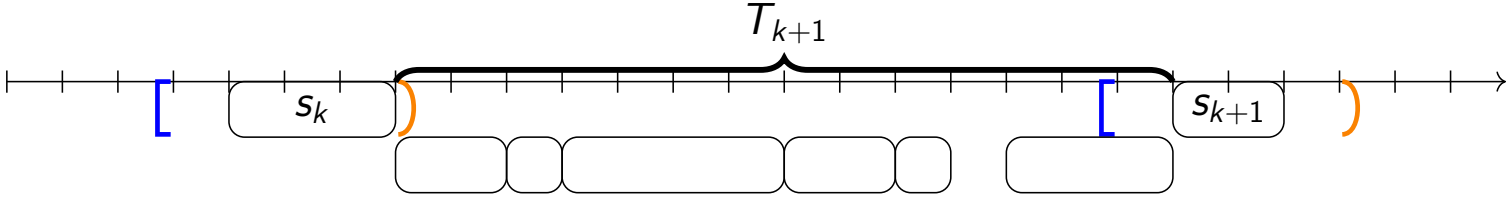

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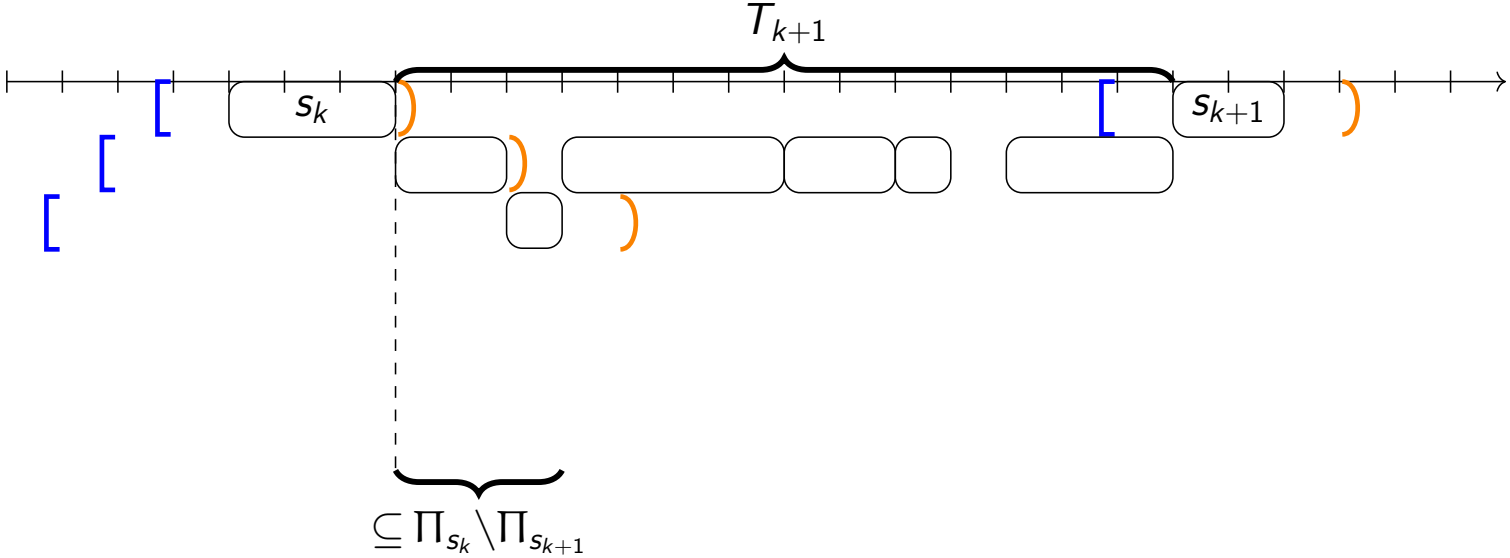

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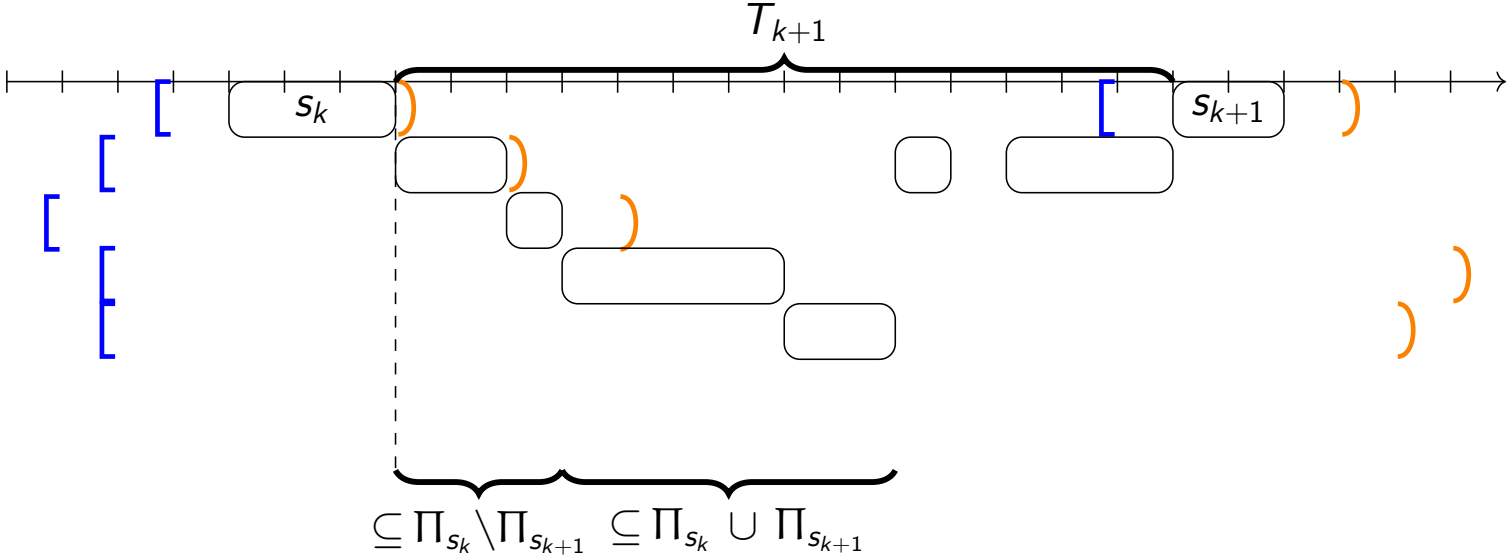

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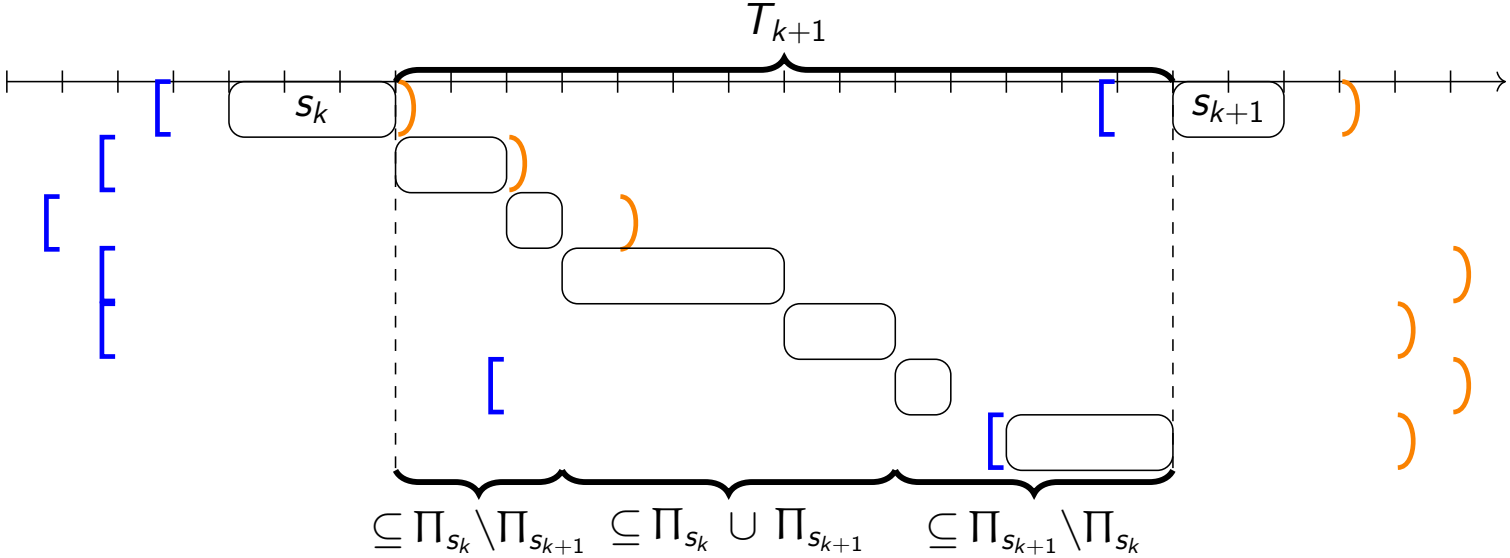

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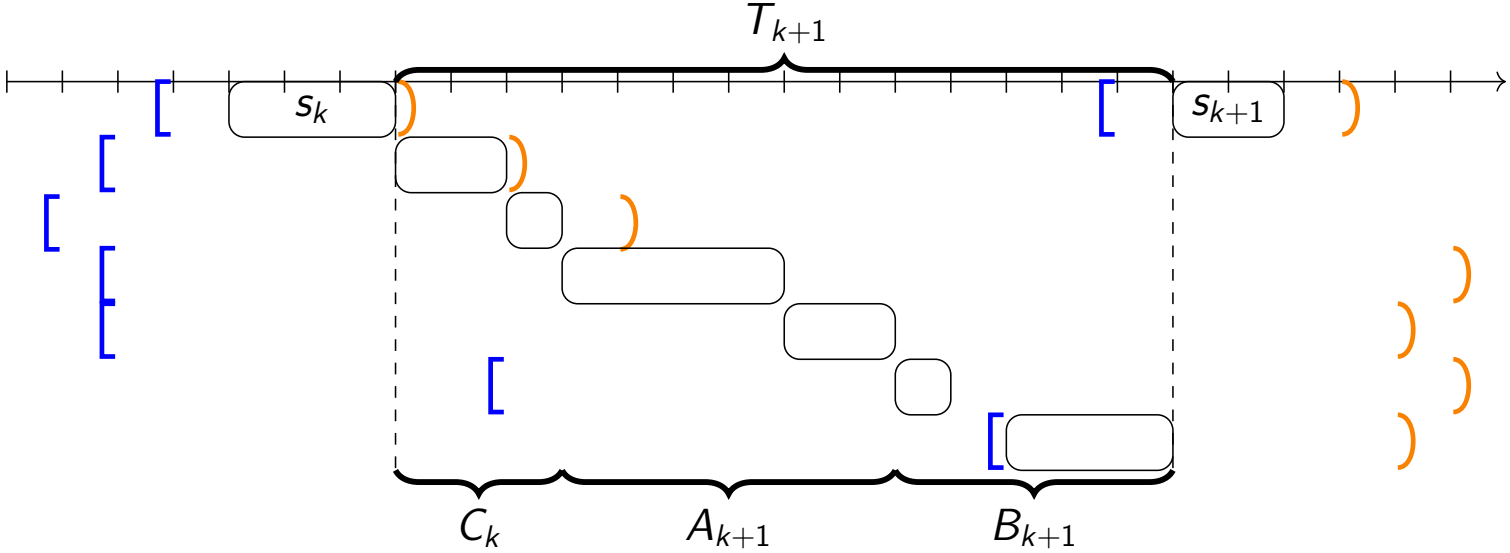

Summit-based dominance (cont.)

Definition: Summit-ordered schedule

A feasible schedule σ is called summit-ordered if it follows the (wED) rule and is of the form $B_1 s_1 \dots s_k C_k A_{k+1} B_{k+1} s_{k+1} \dots s_N C_N$ where:

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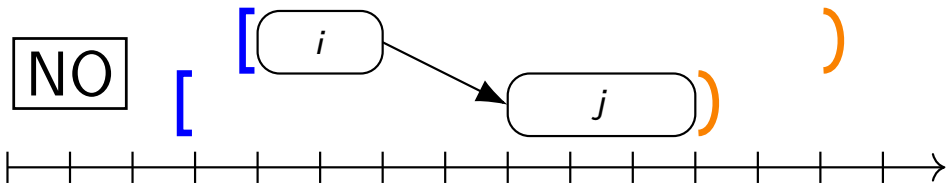
Proposition [Erschler, Fontan, Merce 85]

On $1|r_j, d_j|C_{max}$ the summit-ordered schedules are dominant.

With precedence relations



With precedence relations

- *prec* precedence graph of the instance: "$i \longrightarrow j$".

Definition: *prec*-consistency

An instance I of $1|prec, r_j, d_j|C_{max}$ is called *prec*-consistent when:

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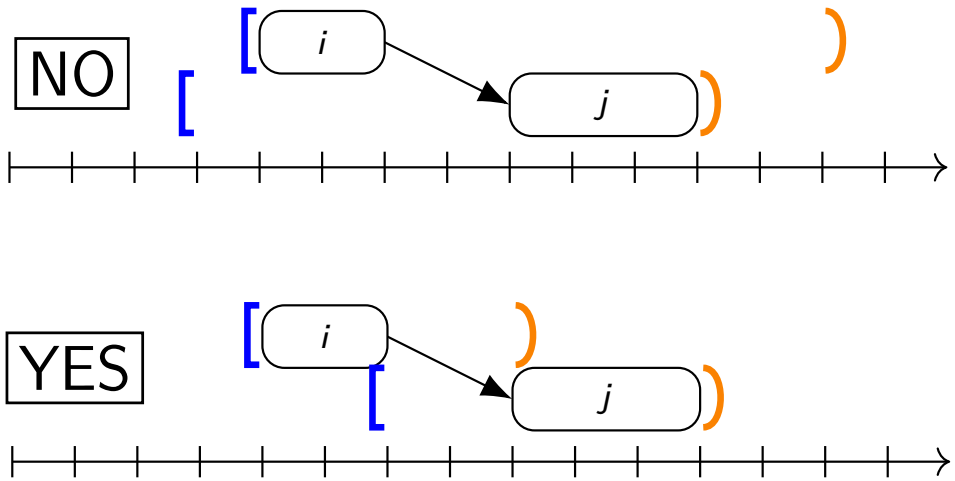

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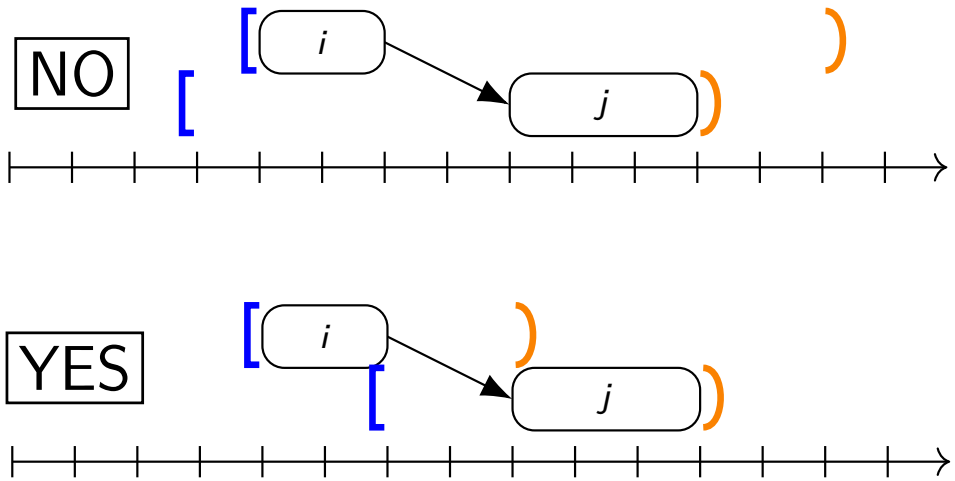
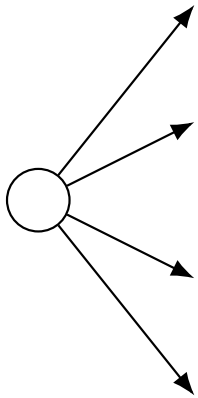


Proposition

On the *prec*-consistent instances of $1|prec, r_j, d_j|C_{max}$ the summit-ordered schedules are dominant.

Dynamic programming with the summit-ordered schedules

s_0



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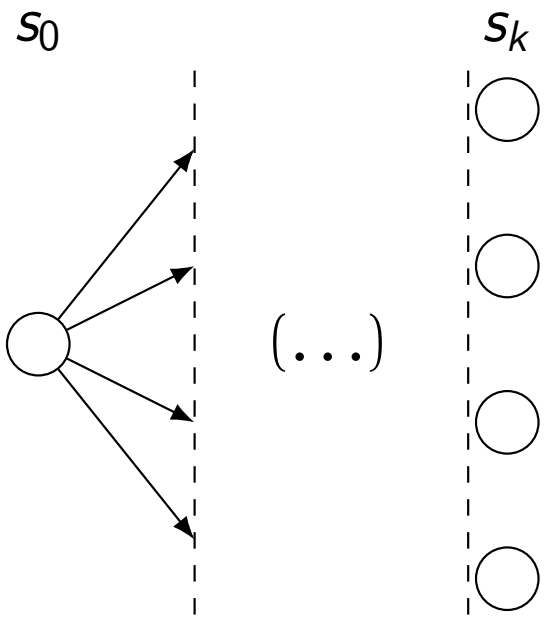

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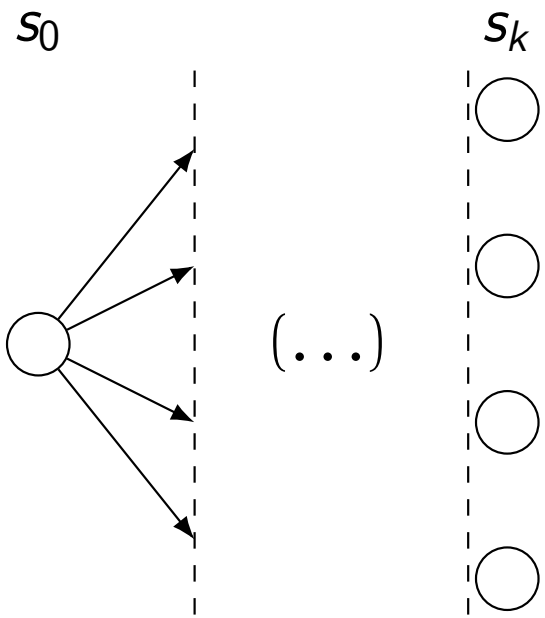

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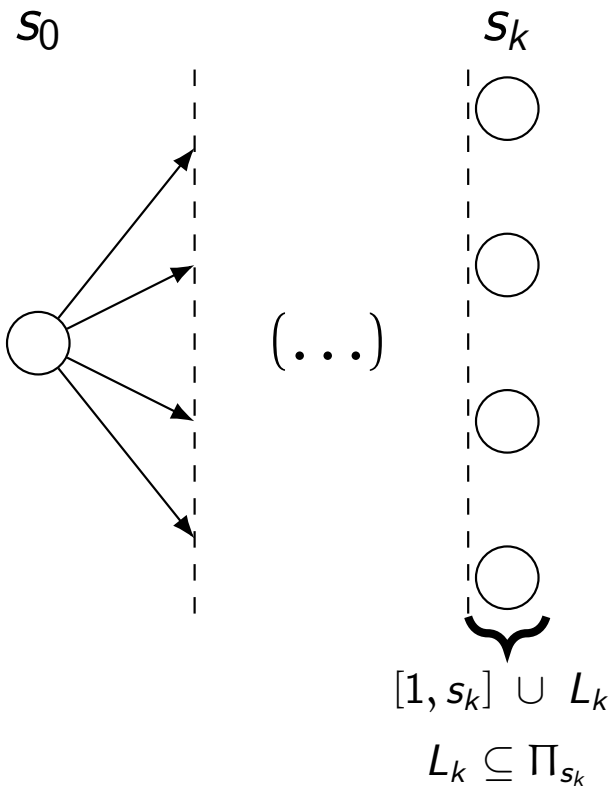

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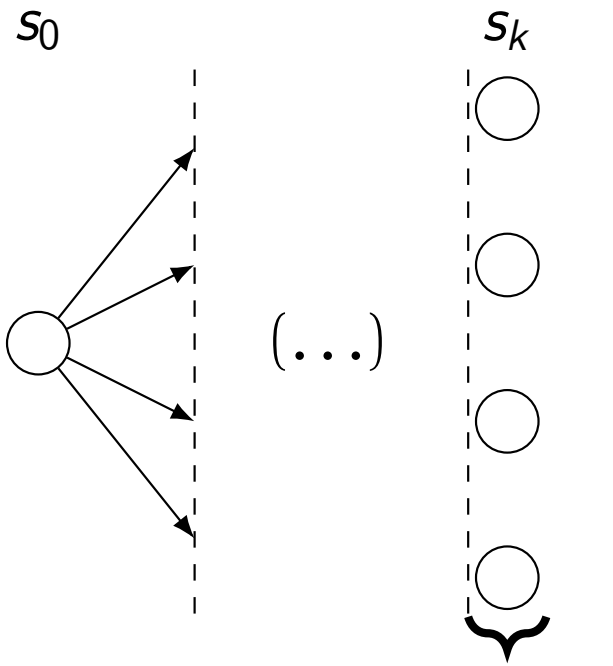

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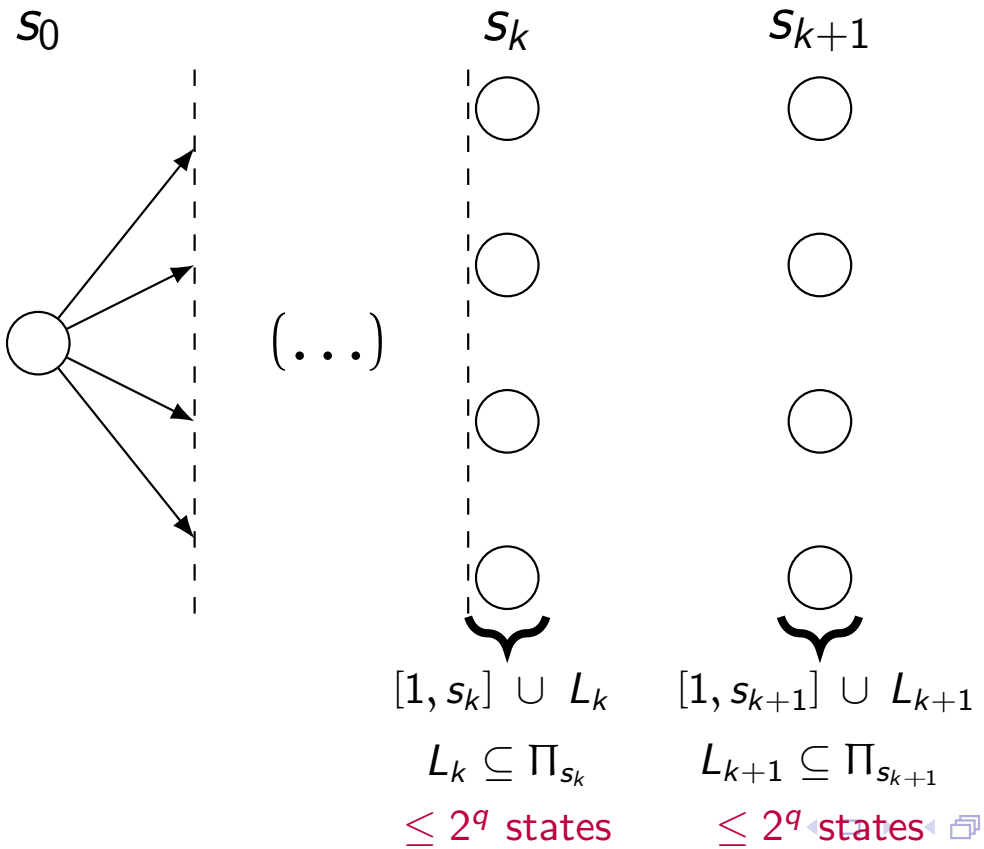

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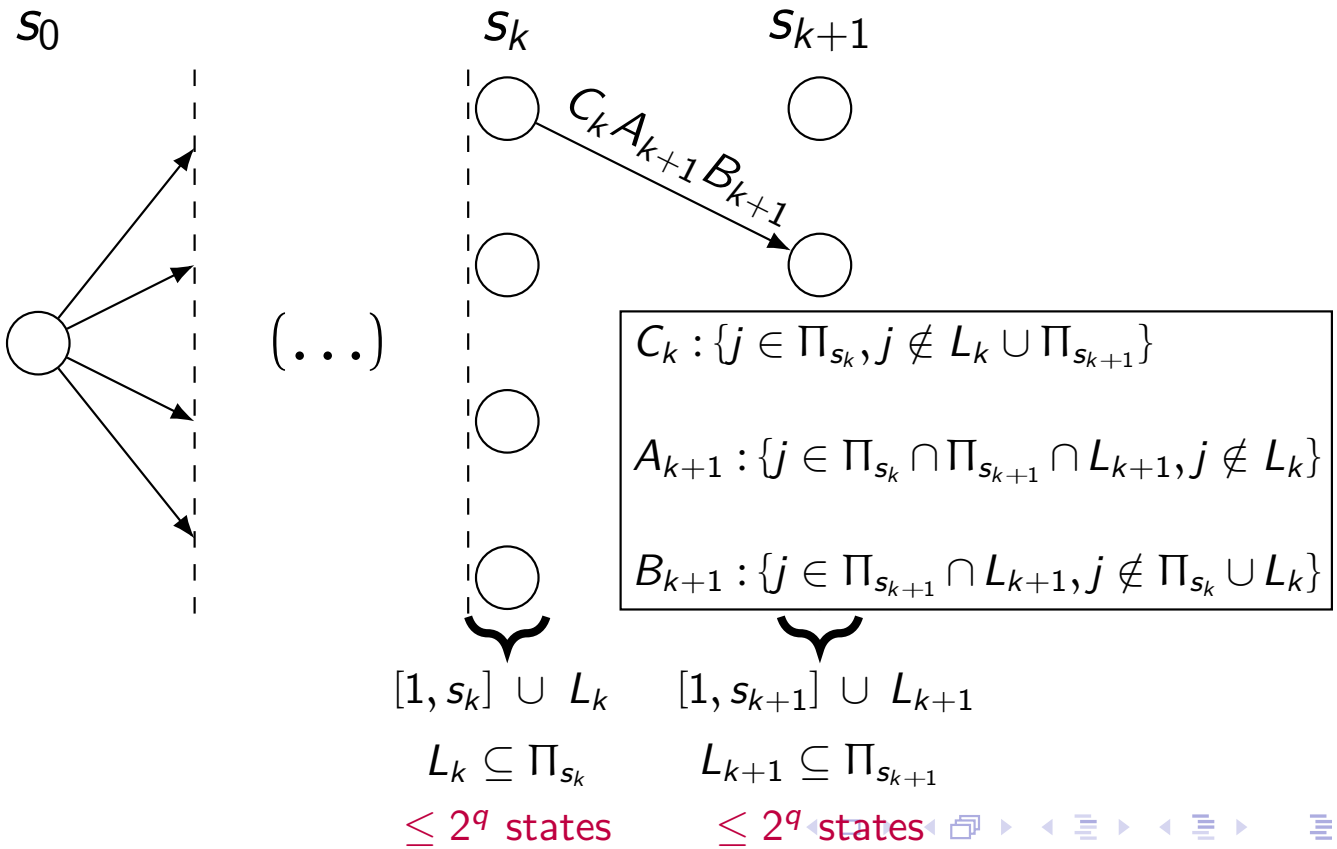

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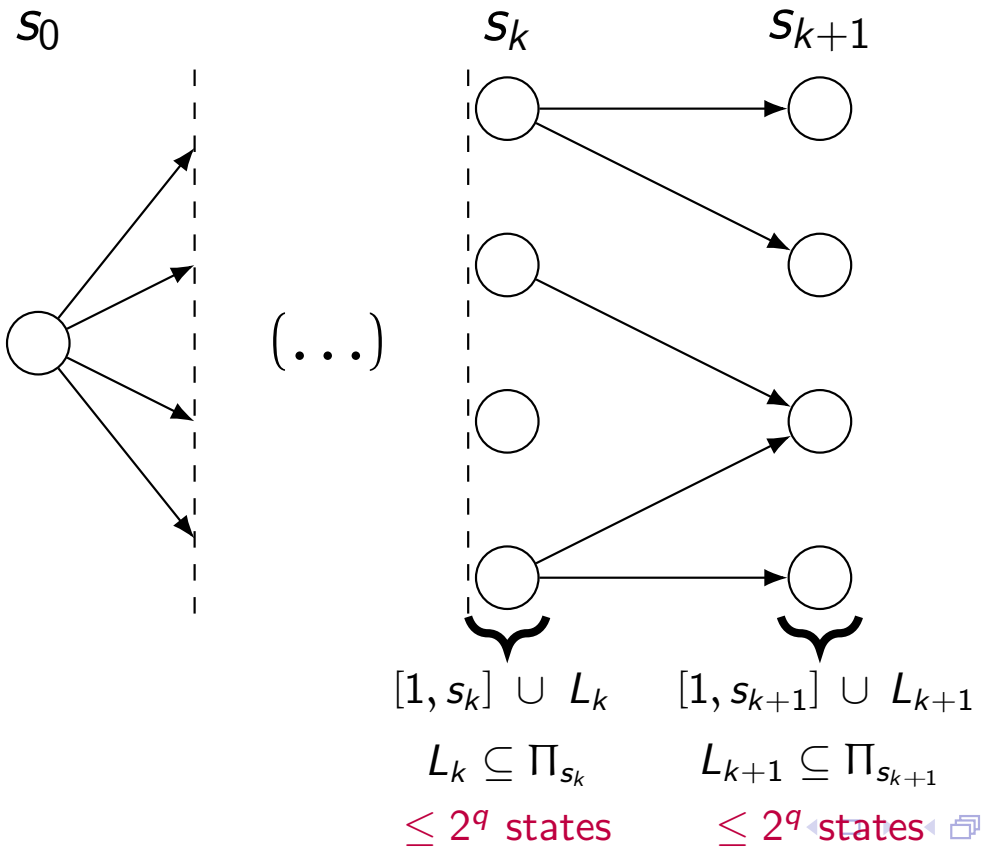

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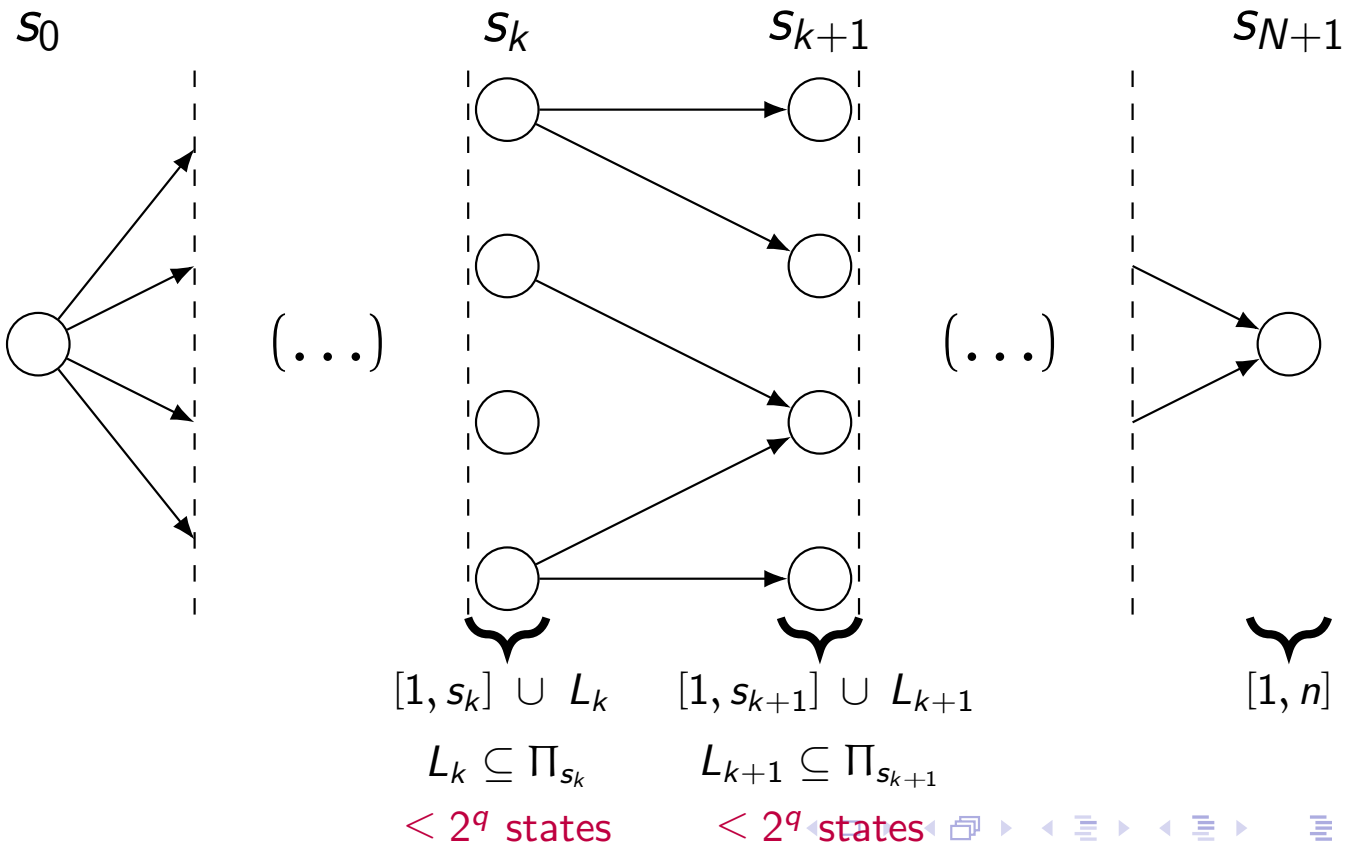

Main result

Theorem [M., Hanen, Munier 24]

$1|prec, r_j, d_j|C_{max}$ can be solved in time:

$$\mathcal{O}(max(1, q^2 \cdot 4^q) \cdot N + n^2)$$

where N is the number of summits ($\leq n$).

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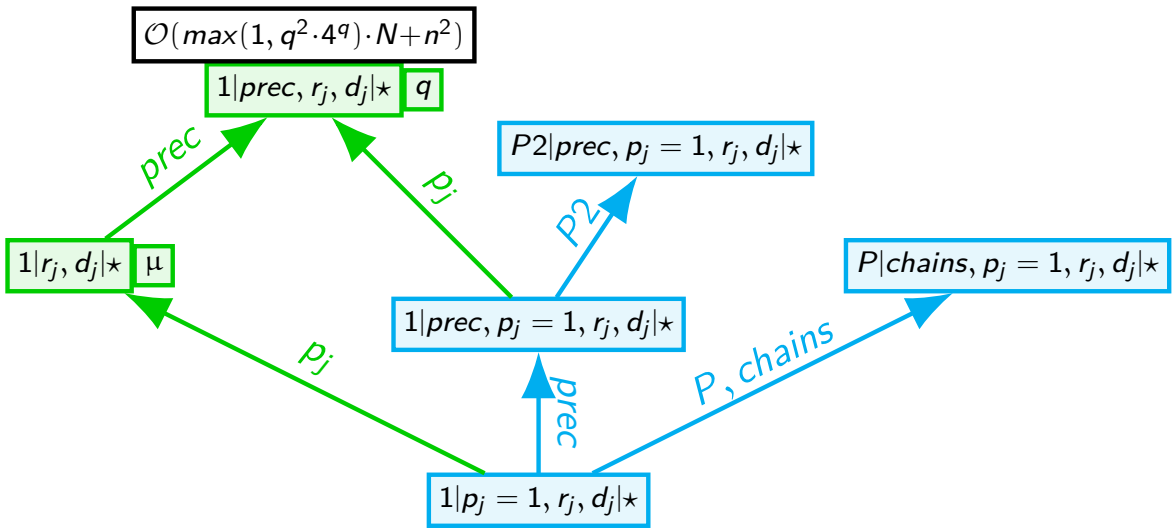
Corollary: Maximum lateness

$1|prec, r_j|L_{max}$ can be solved in time:

$$\mathcal{O}(max(1, q^2 \cdot 4^q) \cdot N \cdot log(D) + n^2)$$

where N is the number of summits ($\leq n$) and $D = (max_{j \in \mathcal{J}} r_j) + (\sum_{j \in \mathcal{J}} p_j)$.

Problem map



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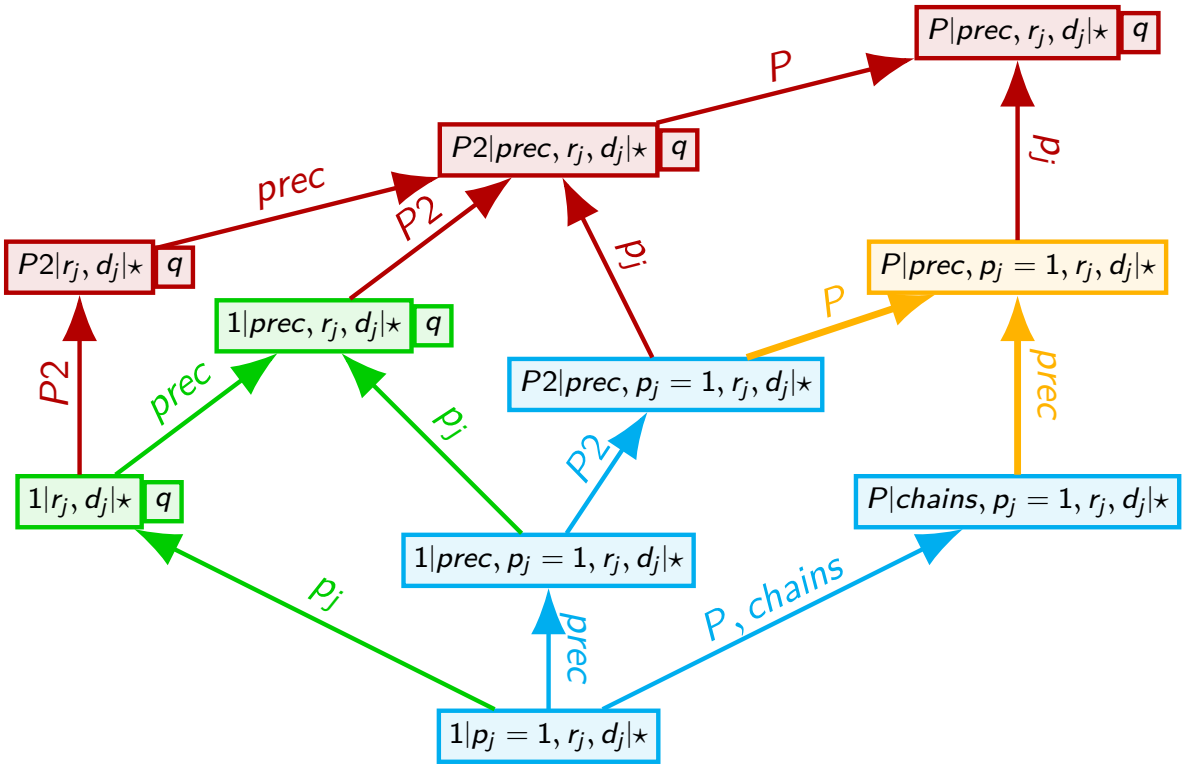

Conclusion & future work

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- New structural parameter on single machine scheduling with job time windows.
- *FPT* algorithm on $1|prec, r_j, d_j|C_{max}$ derived from a new dominance rule.

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Future work

- Consider other settings (like $P|prec, p_j = 1, r_j, d_j|C_{max}$).
- Refine the approach (bounds, heuristics).