Parameterized complexity of single machine scheduling problems with time windows

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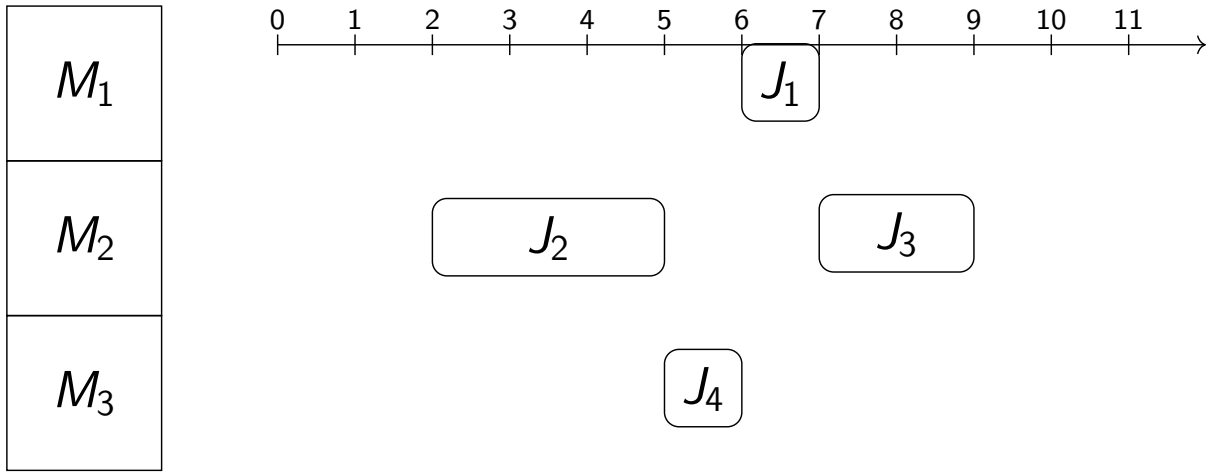
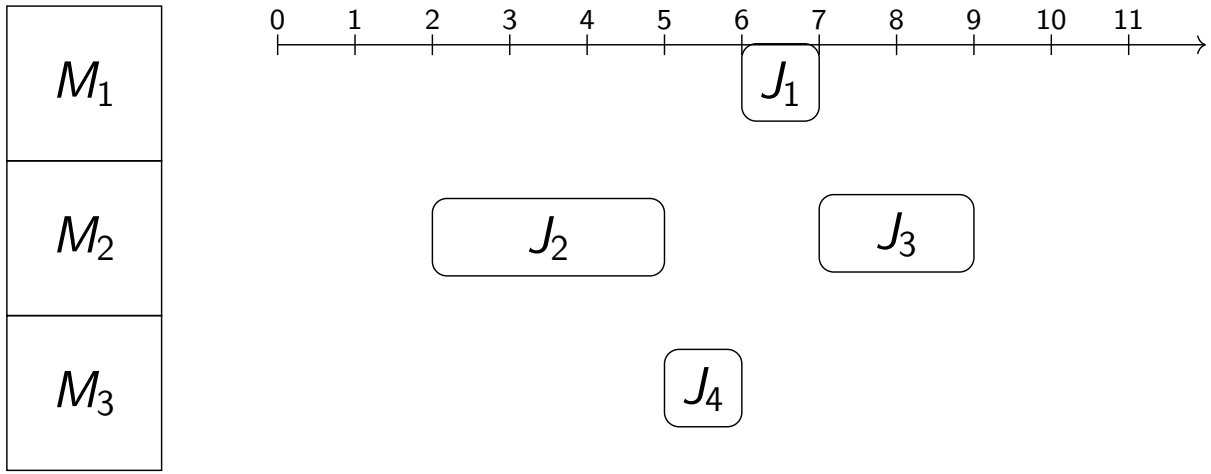
Machine scheduling

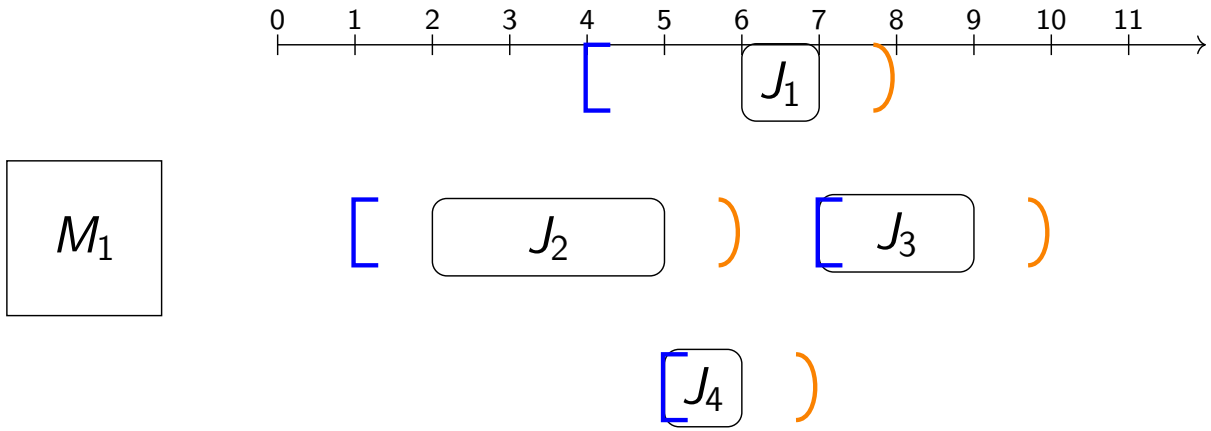


Machine scheduling



machines | job properties | optimization criterion

Machine scheduling



$1|r_j, d_j|\star$

Exploring complexity frontiers

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- Add precedence delays $\ell_{i,j}$

Classical complexity theory

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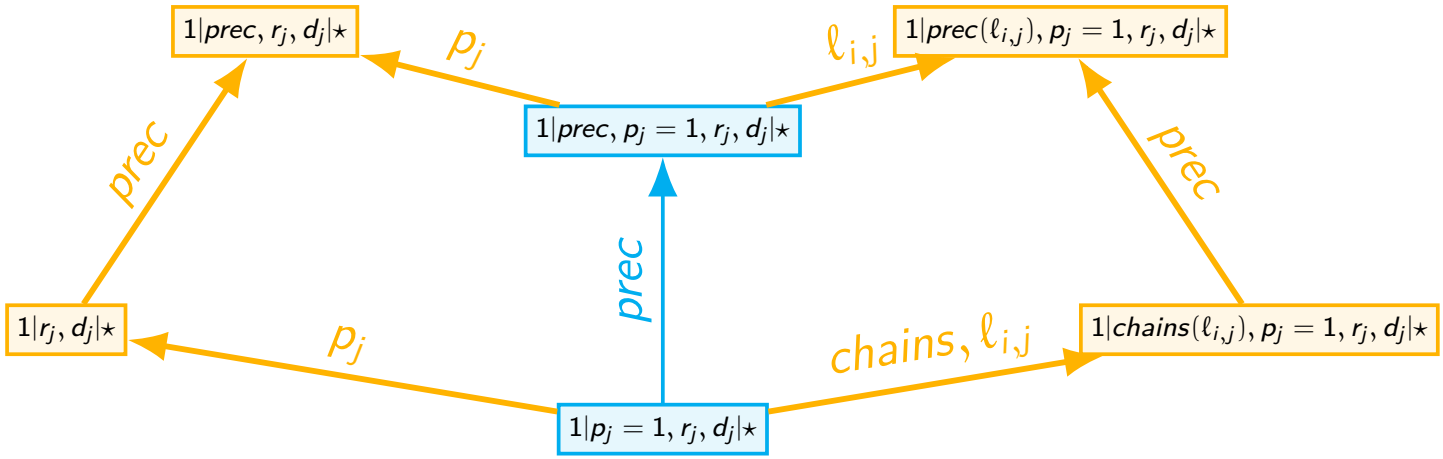
Classical complexity theory



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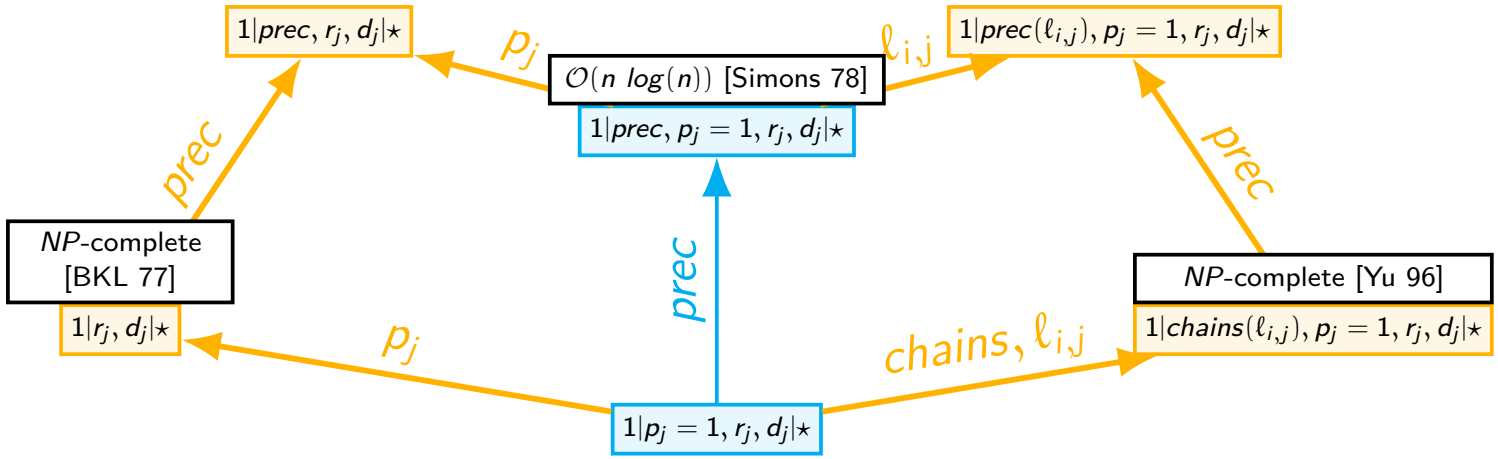
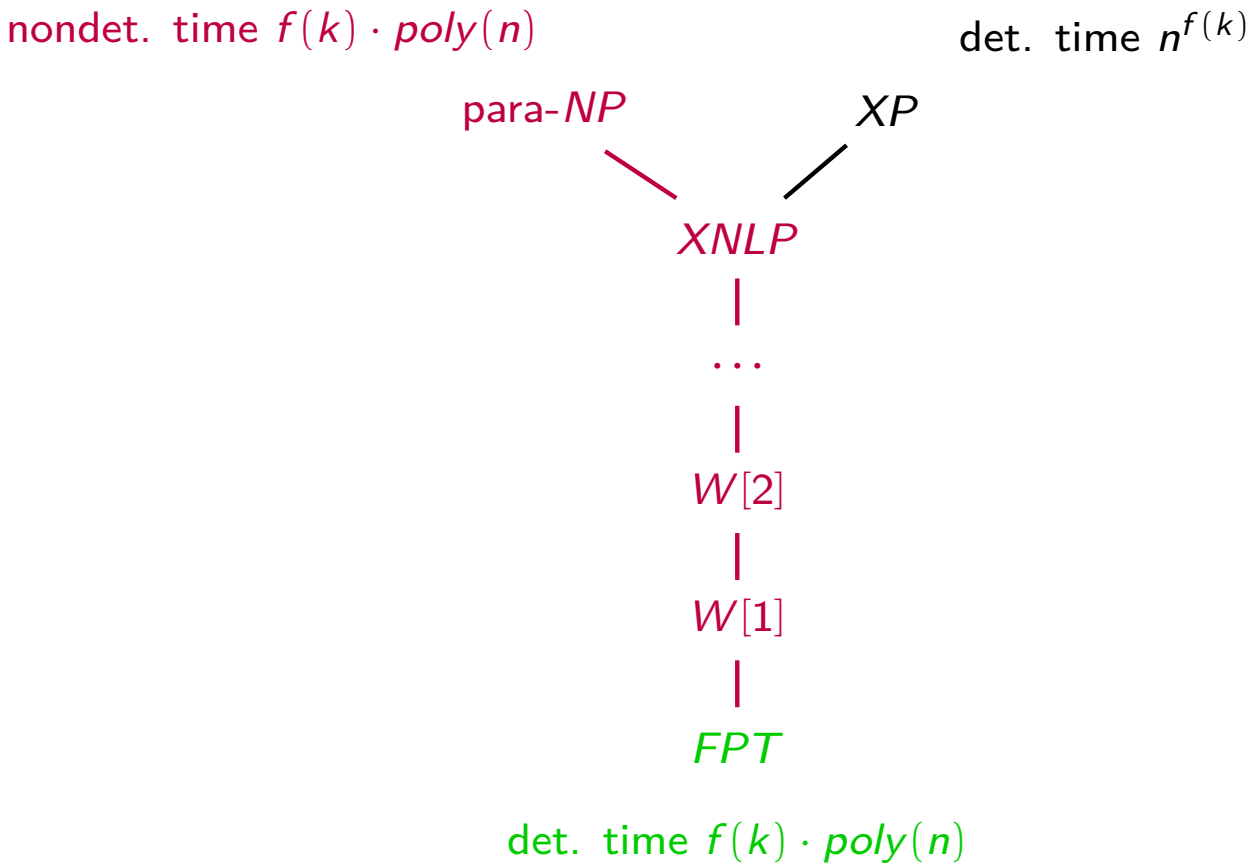


Parameterized complexity classes



Parameterized complexity in machine scheduling

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- Negative results via *FPT* reductions
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    #machines m
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- Mixed Integer Linear Programming
  - [Hermelin, Karhi, Pinedo, Shabtay 2018]
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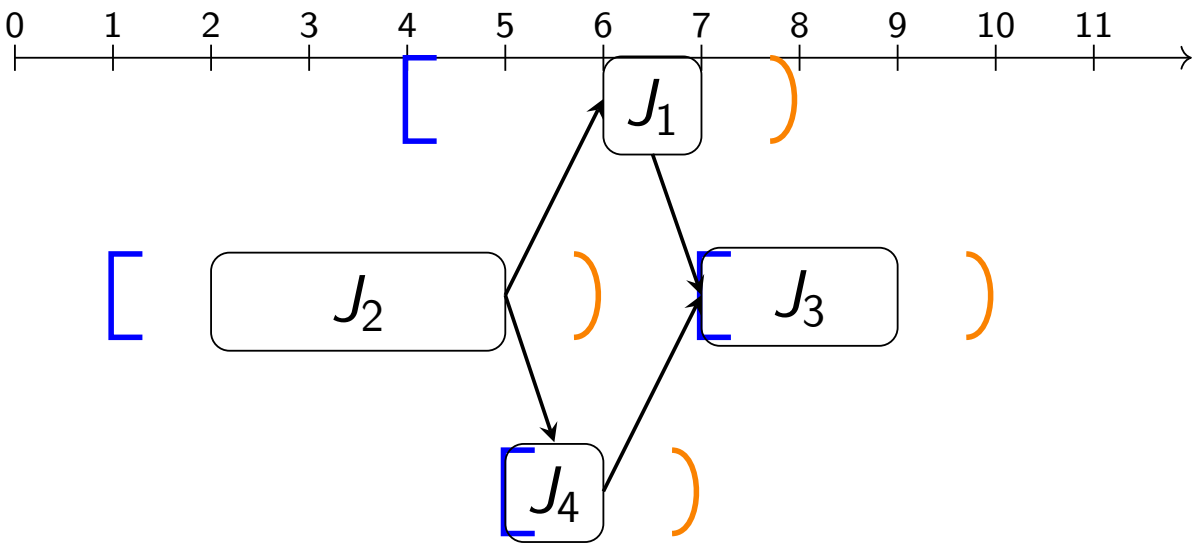
- Dynamic Programming
  - [Baart, He, de Weerdt 2021]
    slack $\sigma = max_j(d_j - r_j - p_j)$
    $1|r_j, s_{jk}, reject, \bar{d}_j|\sum_{j \notin R} w_j T_j + \sum_{j \in R} v_j$ in time $\mathcal{O}(\sigma^3 4^\sigma \cdot n^2)$.

Pathwidth μ



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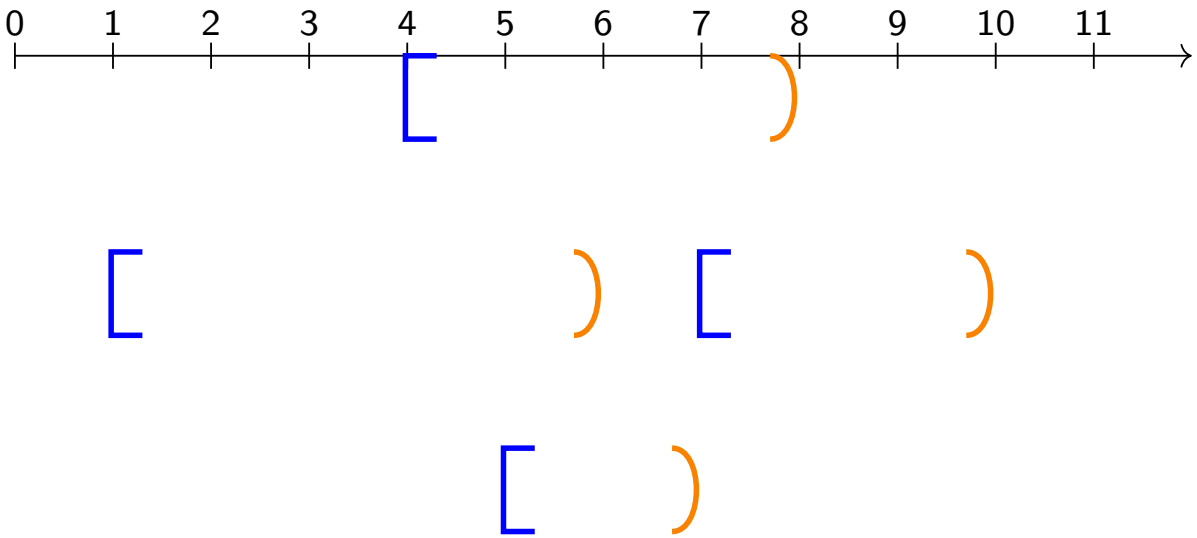

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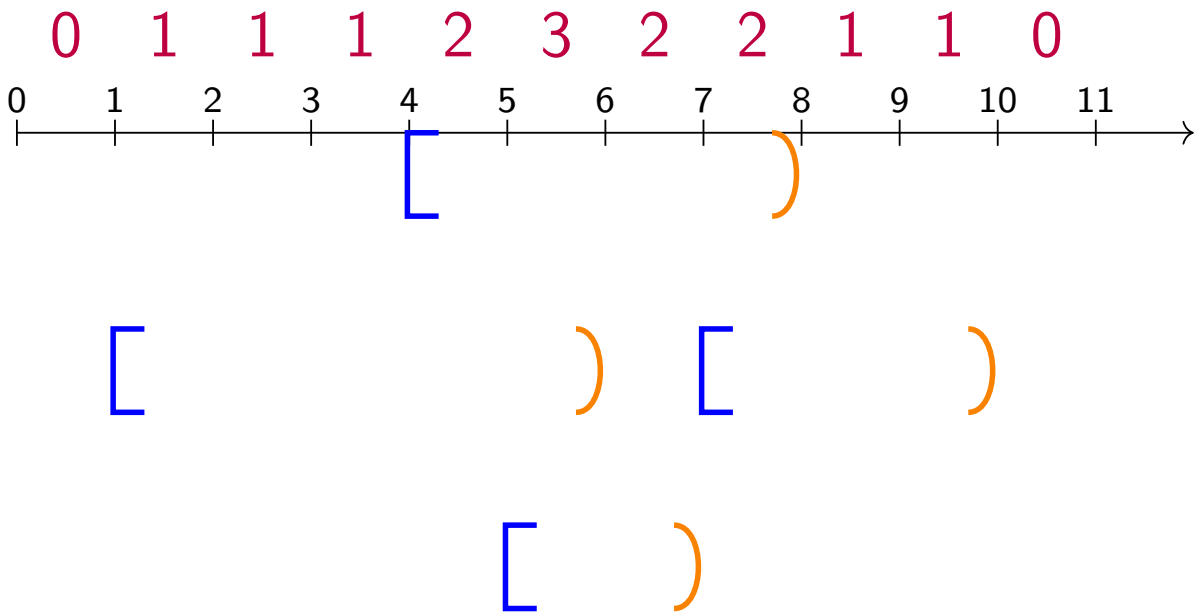

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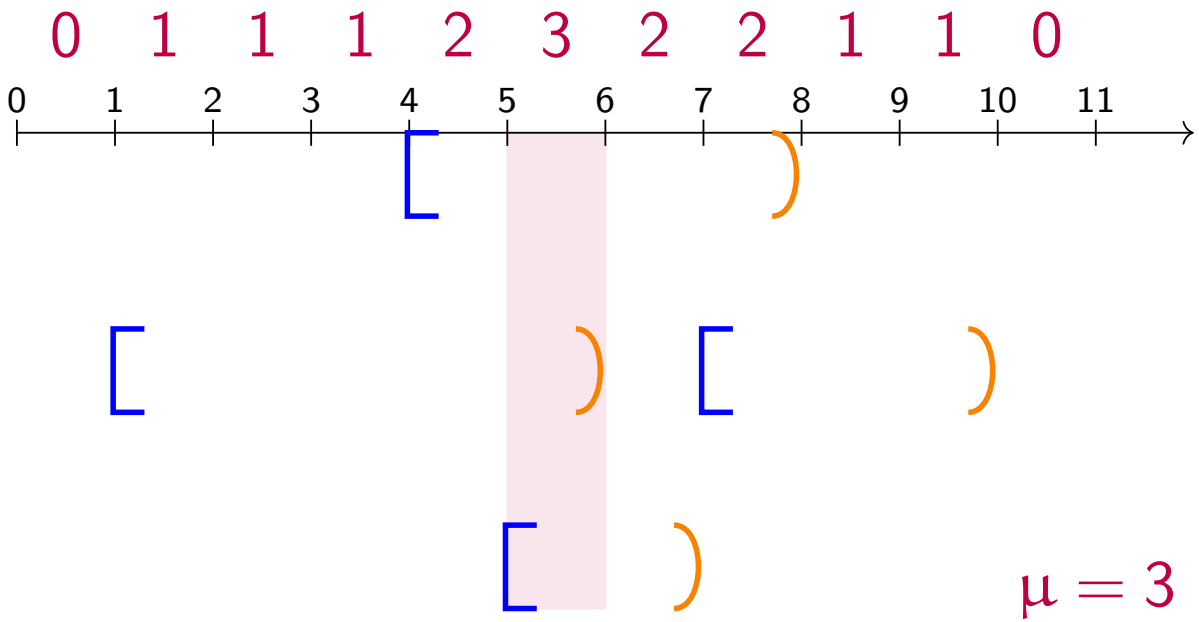

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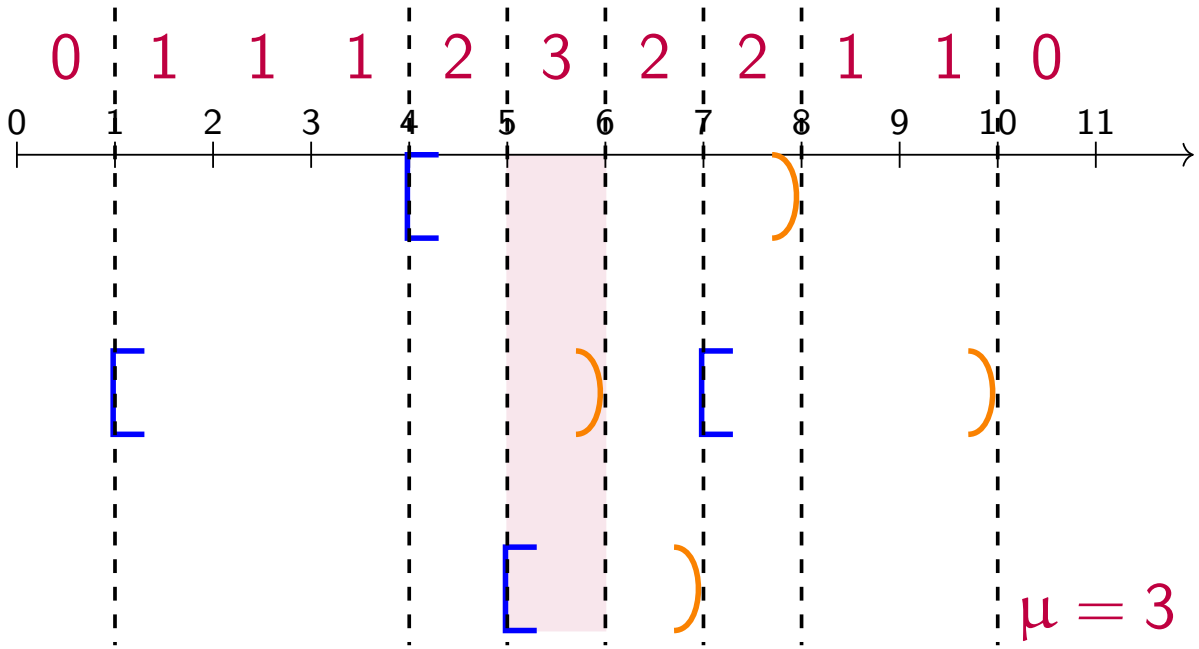
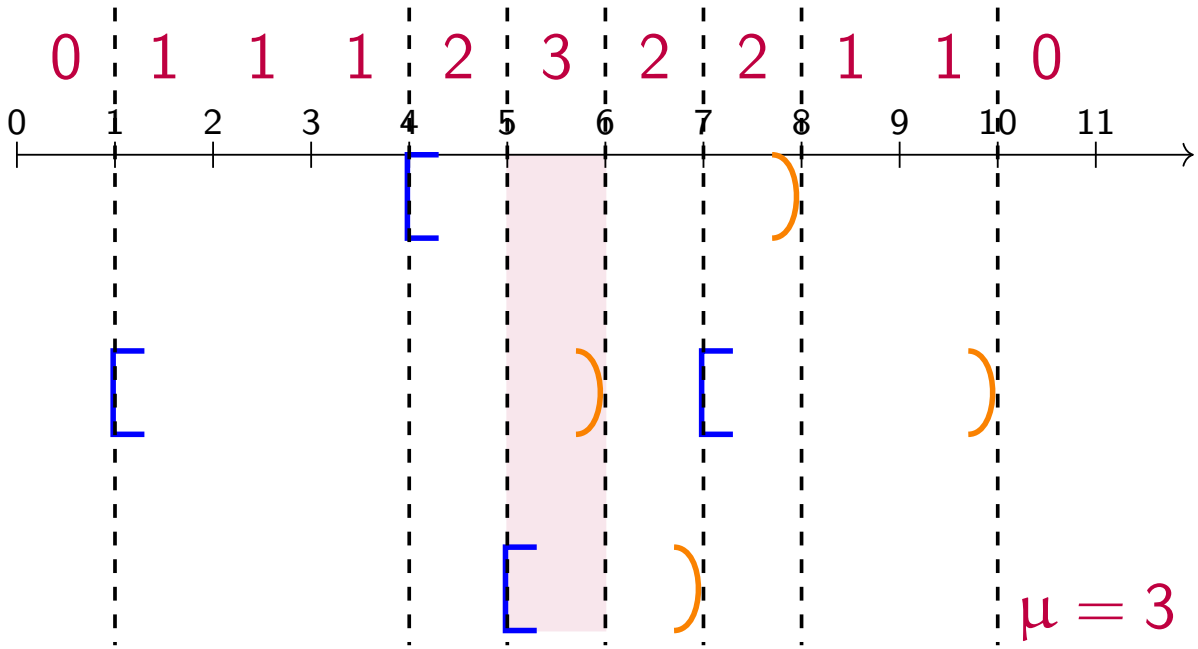
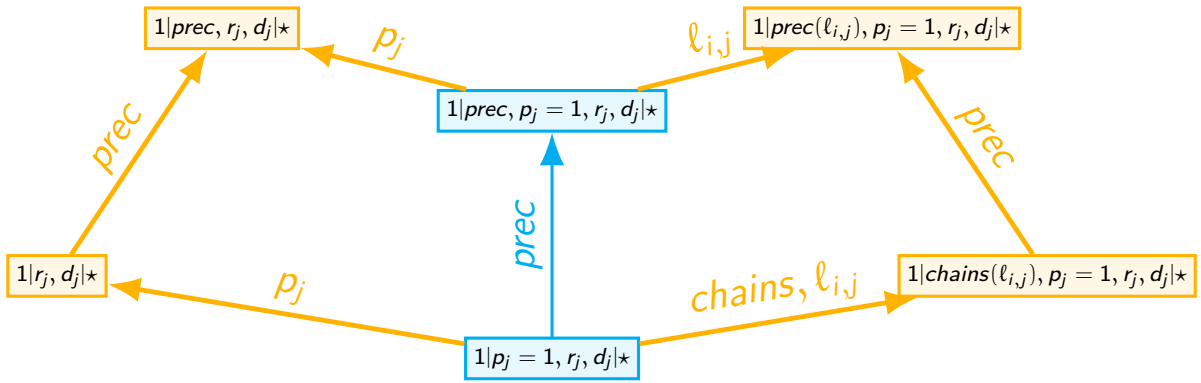


Pathwidth μ



- [MAPSP 2022] $1|prec, r_j, d_j|C_{max}$ is *FPT* parameterized by μ.

Parameter μ



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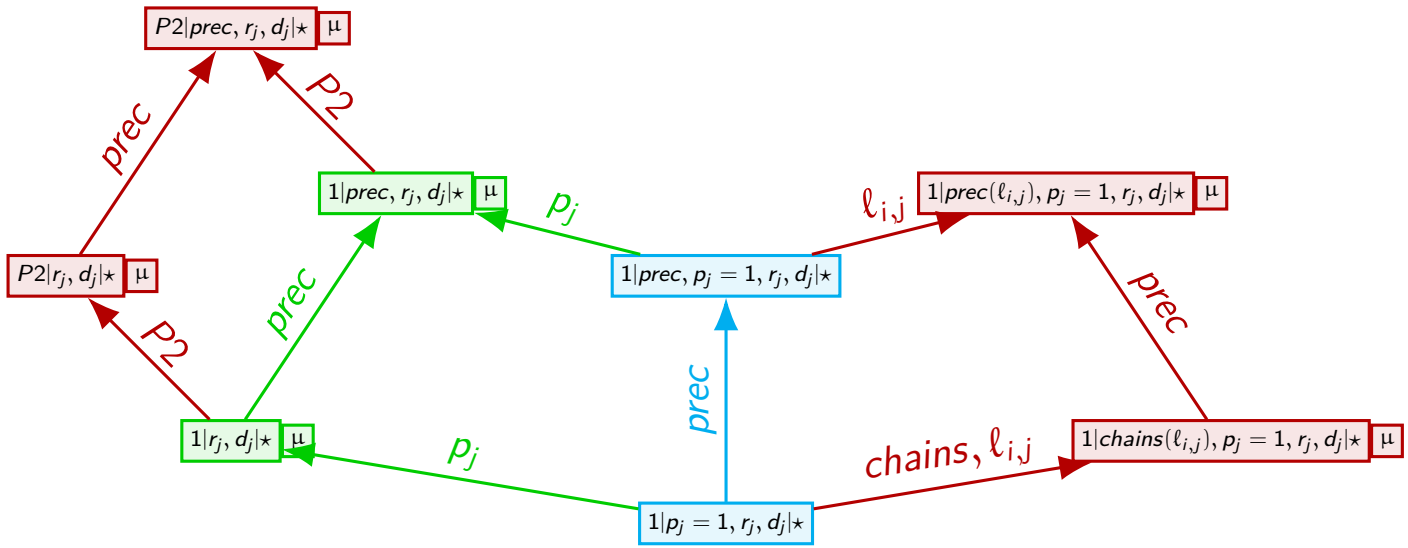

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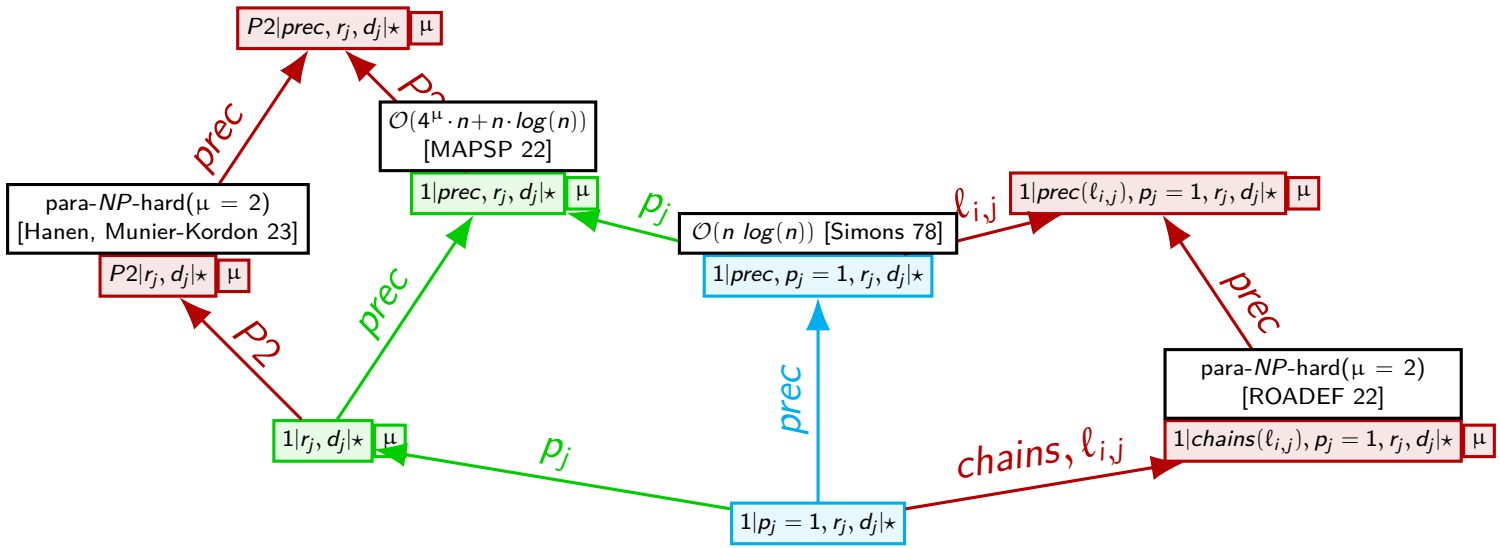
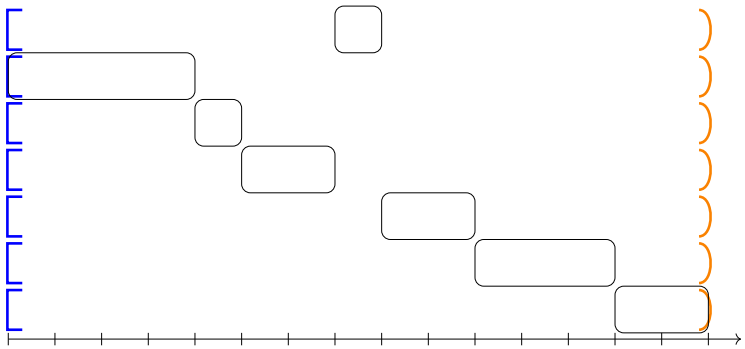
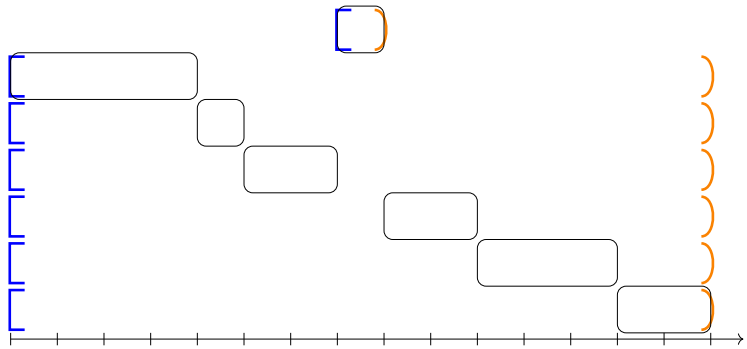


Pathwidth vs q-proper level



$\mu = n$



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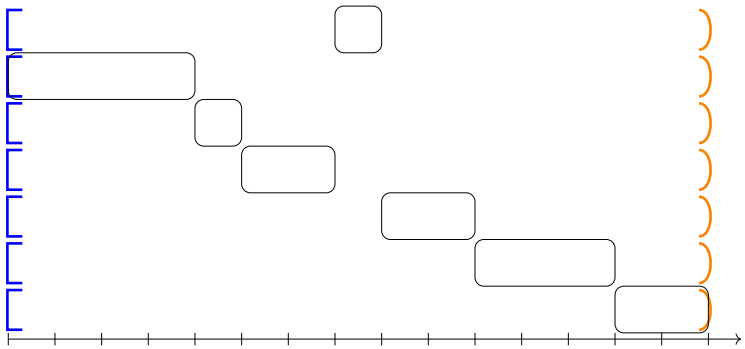

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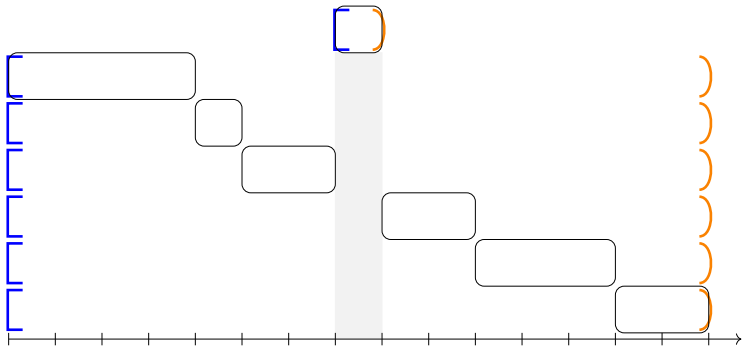

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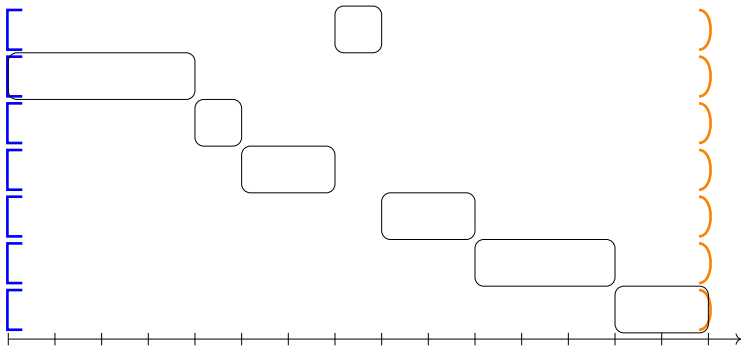

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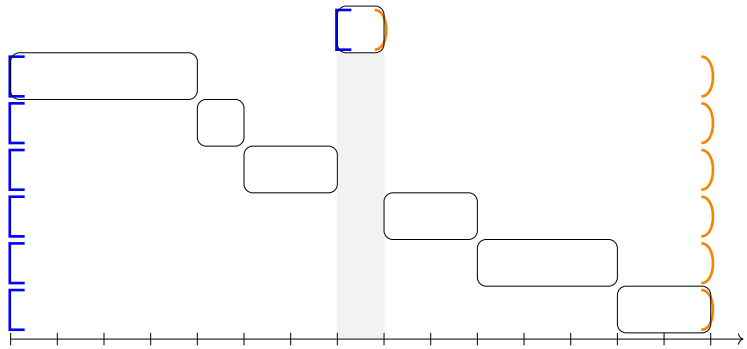

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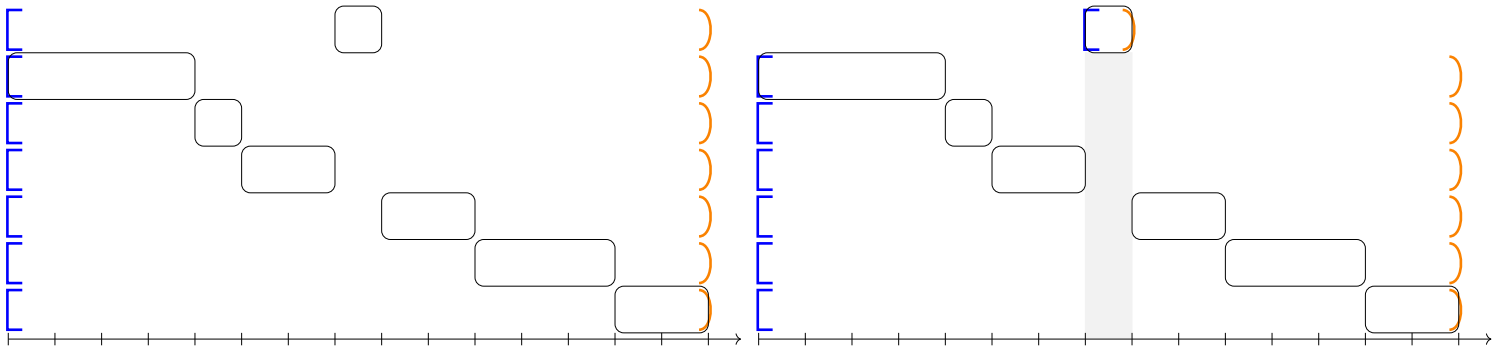
Pathwidth vs q-proper level



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- [ISCO 2024 (submitted)]
  $1|prec, r_j, d_j|C_{max}$ in time $\mathcal{O}(q^2 4^q \cdot n + n \cdot log(n))$.

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  - [Baart, He, de Weerdt 2021]
    $1|r_j, s_{jk}, reject, \bar{d}_j| \sum_{j \notin R} w_j T_j + \sum_{j \in R} v_j$: $(1+\epsilon)$ approximation in time $\mathcal{O}(\mu^2 2^\mu \cdot \frac{n^3}{\epsilon^2})$.