Parameterized complexity of parallel machine scheduling with release dates, deadlines and precedence relations

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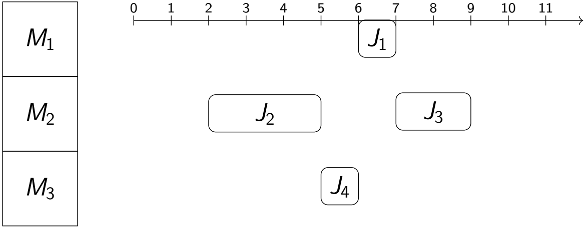
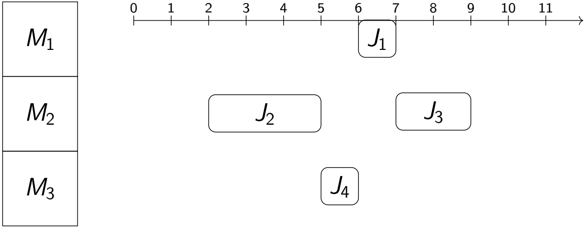
Parallel machine scheduling

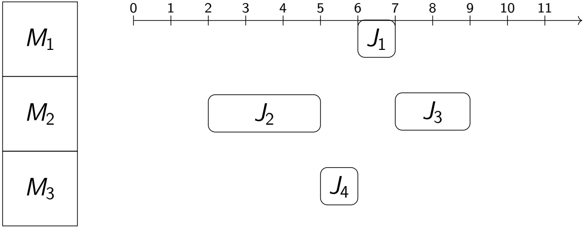


Parallel machine scheduling



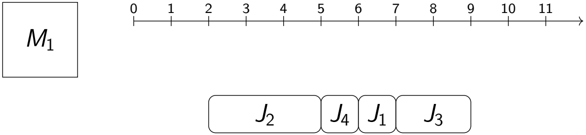
machines | job properties | optimization criterion

Parallel machine scheduling



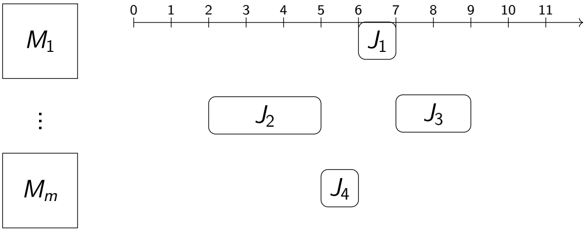
$$P3|.|.$$

Parallel machine scheduling



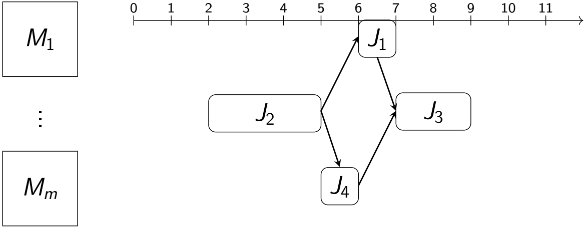
$1|.|.$

Parallel machine scheduling



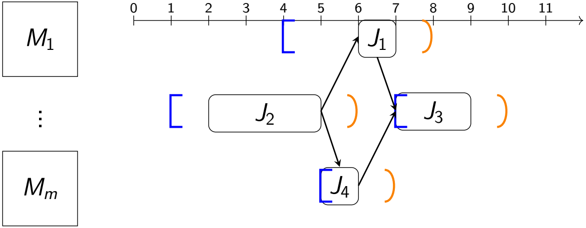
$P|.|.$

Parallel machine scheduling



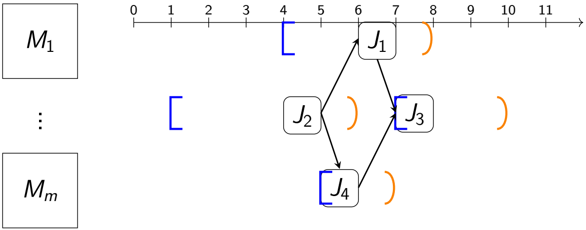
$$P|prec|.$$

Parallel machine scheduling



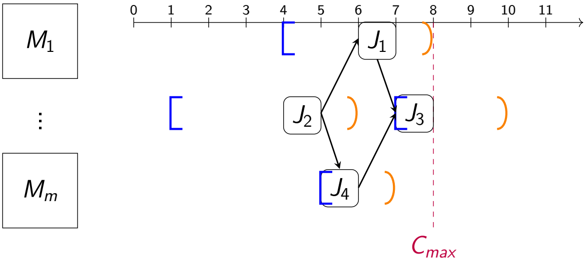
$$P|prec, r_j, d_j|.$$

Parallel machine scheduling

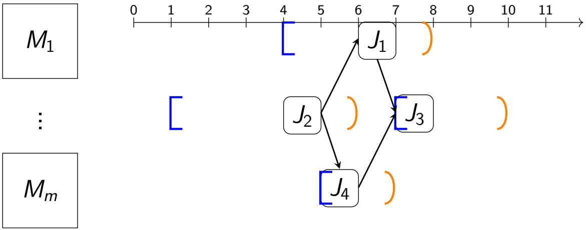


$$P|prec, p_j = p, r_j, d_j|.$$

Parallel machine scheduling



$$P|prec, p_j = p, r_j, d_j|C_{max}$$



$$P|prec, p_j = p, r_j, d_j|\star$$

Typically desired properties

- $1 \longrightarrow P$

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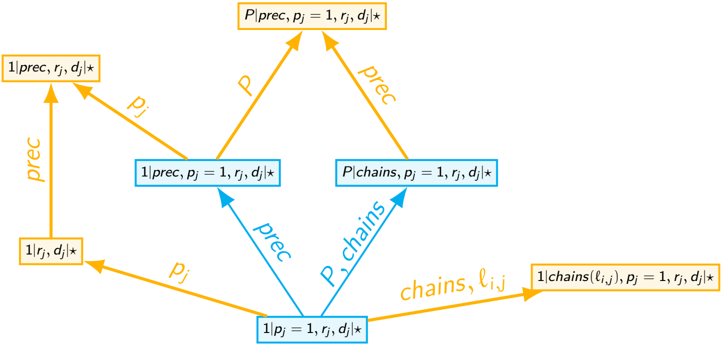
$1|r_j, d_j|\star$ is strongly NP-hard. [Brucker, Kan, Lenstra 1977]

Classical complexity theory

$\boxed{1|p_j = 1, r_j, d_j|\star}$

Classical complexity theory



Going beyond NP-hardness

- P $\subseteq$ NP
- P: deterministic time $poly(n)$
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  - para-NP: nondeterministic time $f(k) \times poly(n)$

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- NP: nondeterministic time $poly(n)$
  - para-NP: nondeterministic time $f(k) \times poly(n)$
- FPT $\subseteq W[1] \subseteq W[2] \subseteq \ldots \subseteq$ para-NP.
  - $W[1] - hard$ = "unlikely to be FPT".

Parameterized complexity in machine scheduling

- [Bodlaender, Fellows 1995]
  - #machines m: $P|prec, p_j = 1|C_{max}$ is W[2]-hard.

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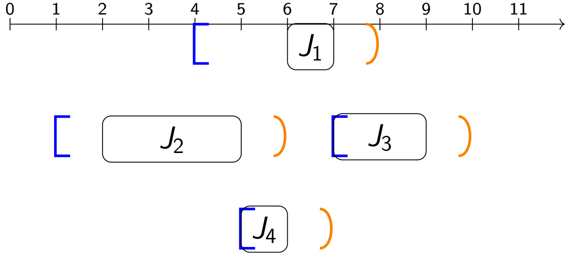
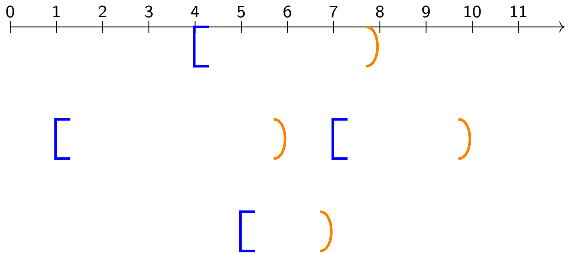
- [Hermelin, Karhi, Pinedo, Shabtay 2018]
  - #distinct d_j, #distinct p_j, #distinct w_j: $1||\sum w_j U_j \in$ FPT with two of the three.

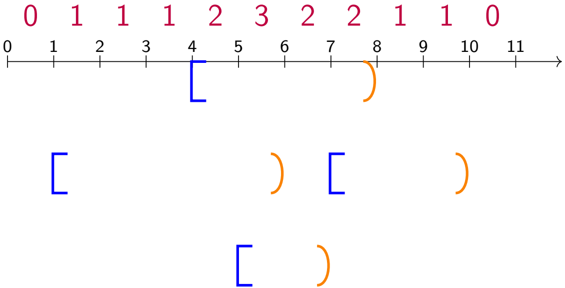
Pathwidth pw



Pathwidth *pw*



Pathwidth pw



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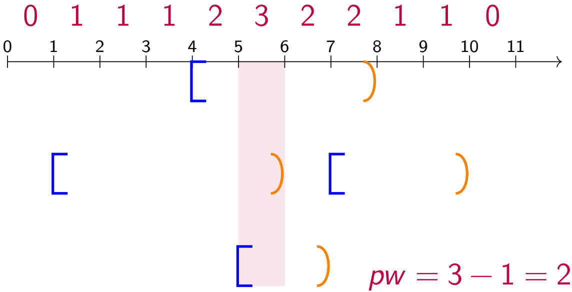
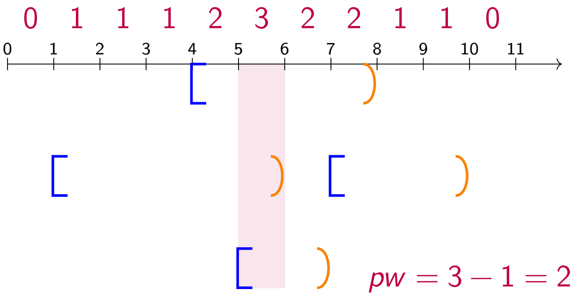


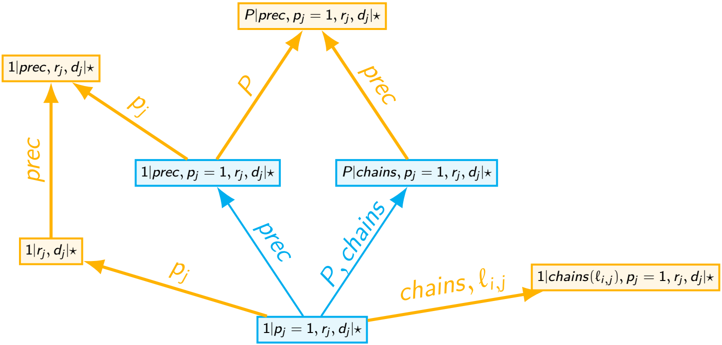
$$pw = 3 - 1 = 2$$

Pathwidth pw



- [Munier 2021] $P|prec, p_j = 1, r_j|L_{max}$ is FPT parameterized by pw.

Parameter *pw*



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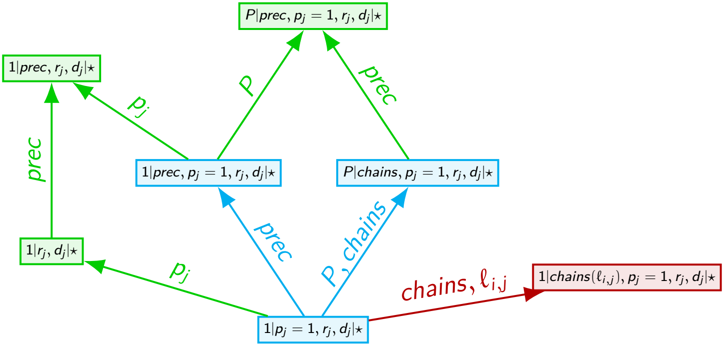
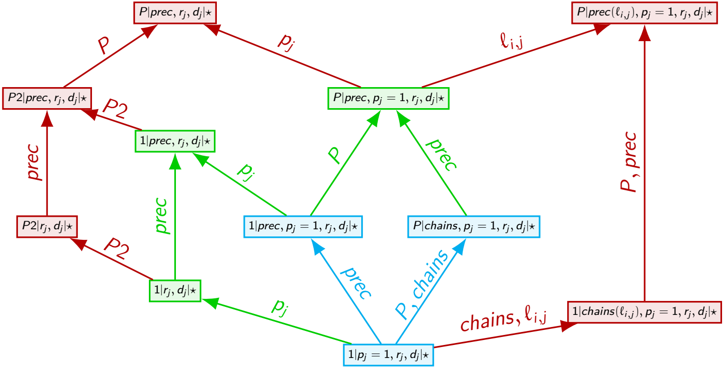
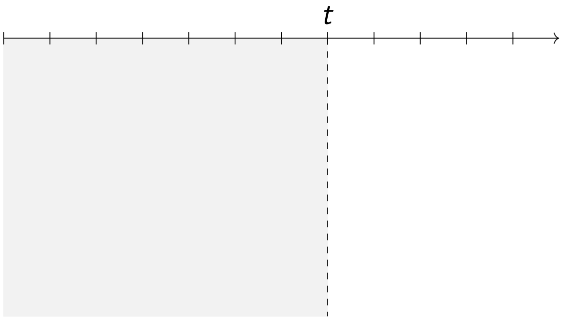


Parameter *pw*



$P|prec(\ell_{i,j}), p_j = 1, r_j, d_j|\star$ is FPT parameterized by $pw + \ell_{max}$

- Dynamic programming algorithm on active schedules



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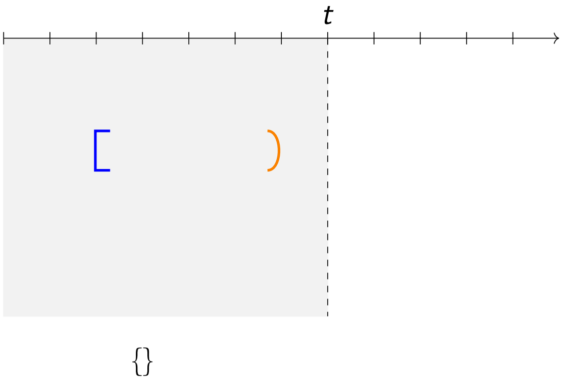

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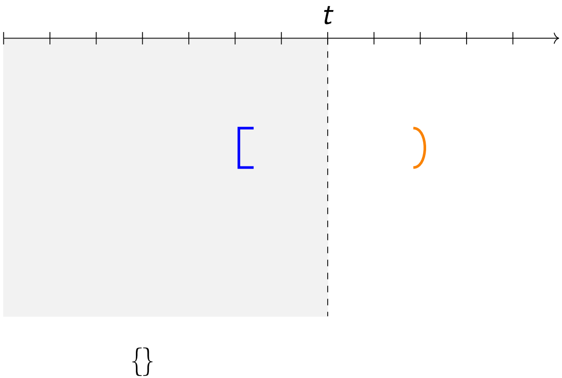

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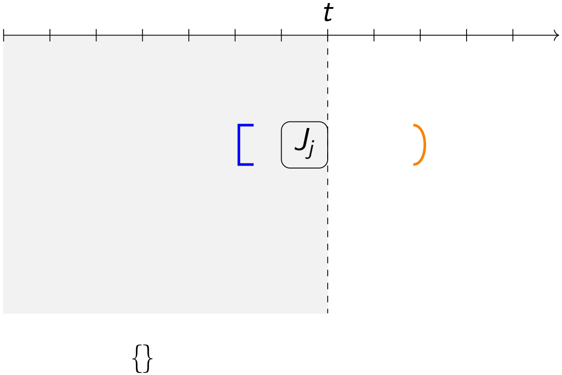

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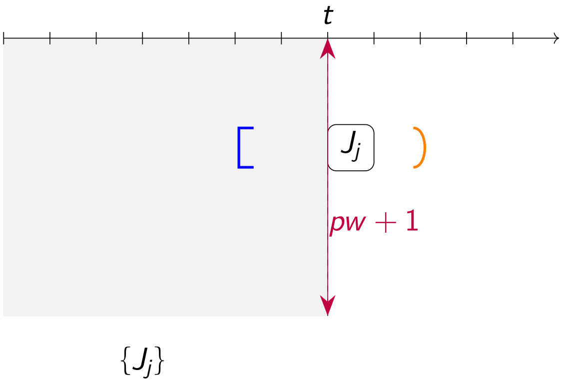

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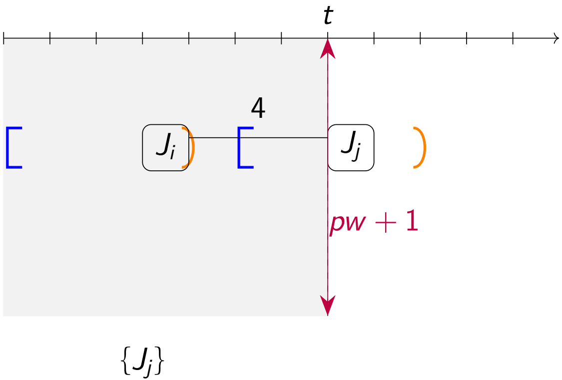

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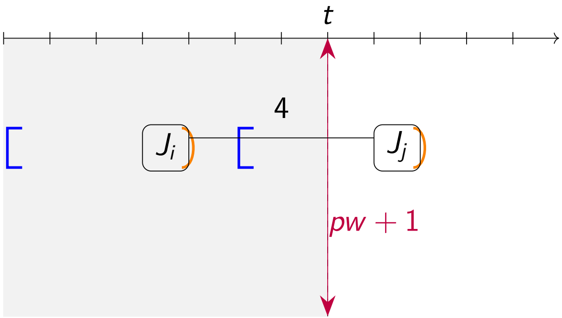

$\{J_j, J_i\}$

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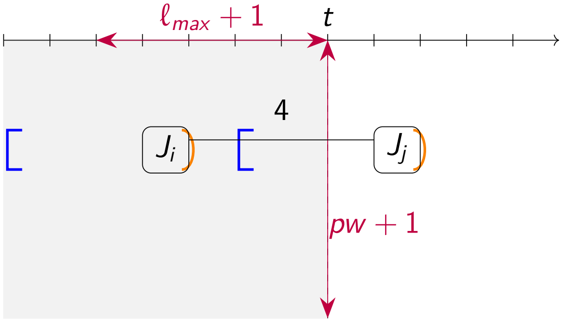
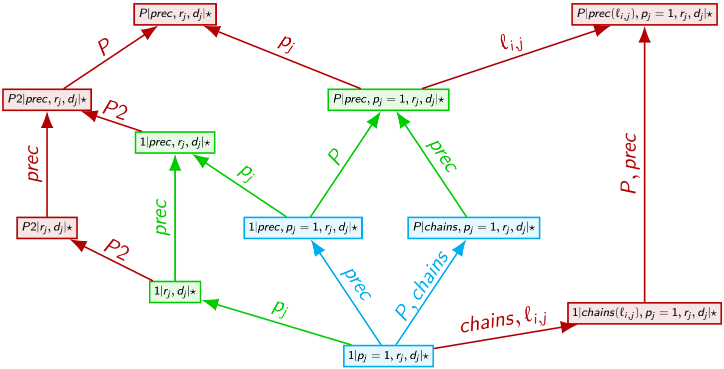


$$|\{J_j, J_i\}| = \mathcal{O}(pw \times \ell_{max})$$

Parameter $pw + \ell_{max}$



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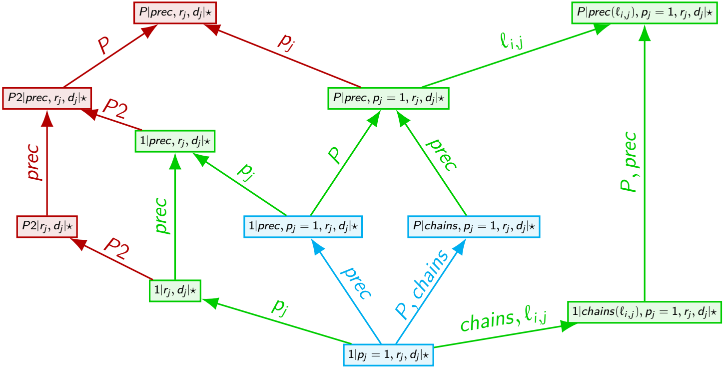

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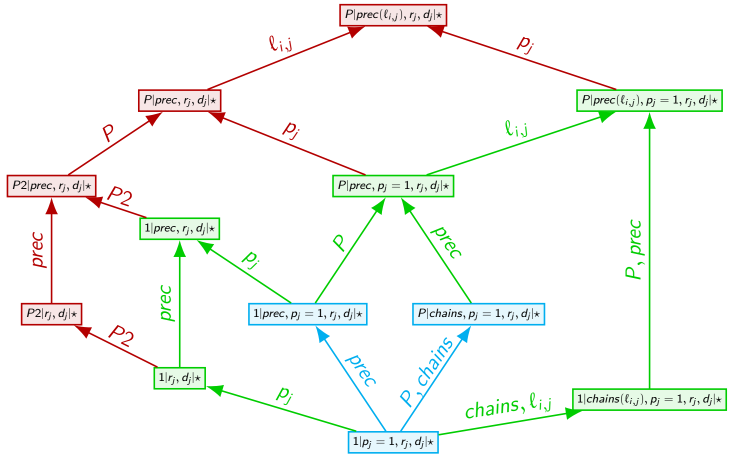

Conclusion

- In-depth classification of scheduling problems (beyond P vs NP).
- More instance families with efficient exact methods.
- Scheduling Zoo [Dürr et al. 2006-Present]